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CONTROLLING A CHAOTIC SYSTEM THROUGH CONTROL PARAMETER SELF-MODULATION

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Chaotic bahaviour, that is, the apparently irregular manifestation of an otherwise strictly deterministic dynamics, is known to be a quite common possibility for many natural systems arising in such varied fields as electromechanics [1-2], quantum optics [3-6], fluid dynamics [7-9], plasma physics [10-12], electronics [13-15] and others. In some instances, however, this type of behaviour is to be avoided, and the problem naturally arises as to how to control it. Since the publication of a seminal work by Ott, Grebogi and Yorke (OGY) [16] it has been realized that chaos can be efficiently controlled by making only small, carefully selected perturbations to a control parameter. A key point for obtaining such control is that a chaotic trajectory has a wealth of *unstable* periodic trajectories embedded on it, together with the sensitive dependence on initial conditions characteristic of chaos [17]. Further details of this method are given in the current literature [18-20], and it has been applied successfully in a number of experimental situations, showing its great potentialities [21-23].

Another approach to the control of chaotic systems, which is closely related to the OGY method, is what is known as the technique of Occasional Proportional Feedback (OPF) [24-26]. It consists in monitoring and sampling an output signal from the system and feeding it back, with appropiate offset and gain, to modify a control parameter during selected time intervals.

Let us consider that the physical system is described by a set of ordinary differential equations

$$\begin{aligned} &\dot{x}_i = f_i(x_1, x_2, \ldots, x_n; \alpha_1, \alpha_2, \ldots, \alpha_m; t) \\ &x_i(0) = x_{i0}, i = 1, 2, \ldots, n \end{aligned} \qquad (1).$$

Then, control is obtained by increasing the number of available control parameters and by changing the form of the underlying equations because of the transformation

$$\alpha_k \to \alpha_k + f_{ck}(t, x_j, T, \vec{p}) \tag{2}$$

where α_k denotes the parameter which is varied and f_{ck} is a function with a rather general form, depending *explicitly* on time through the duration of the interval on which control is applied. x_j, T and **p** are respectively a system variable, the sampling time and the new control parameters (like offset and gain, see for exemple [26]). It is clear that even if (1) is an autonomous set of equations, adding (2) will change it to a nonautonomous system and thus the physical phase space will change accordingly. One of the advantages of OPF versus OGY is that it is perhaps more easy to implement experimentally, as no computer control is neccesary [24].

Another method has been proposed recently differing from OPF in some points [27]; in it a *forcing term* of the form

$$F = g(x_j(t-\tau) - x_j(t)) \tag{3}$$

is added to one of the equations (1) with τ being a variable delay. By sweeping τ over a suitable range of values, a number of unstable periodic orbits embedded in the chaotic attractor can be made stable [27]. This technique allows a fairly simple experimental implementation but, correspondingly, the theoretical description is more involved, due to the fact that one has to deal, generally speaking, with systems governed by delay-differential equations, i.e., infinite-dimensional systems [27].

The method we propose in this Letter consists in modifying a control parameter with a non-delayed perturbation directly proportional to an output signal from the system. The method is tested on model equations (4), representing the coupling of two van der Pol oscillators.

$$\begin{aligned}
&\dot{x} = y \\
&\dot{y} = (\varepsilon_1 - (x+\beta z)^2)y - (x+\beta z) \\
&\dot{z} = v \\
&\dot{v} = (\varepsilon_2 - (z+\alpha x)^2)v - (z+\alpha x)
\end{aligned} \qquad (4).$$

This system has been numerically studied elsewhere and has shown to have a rather rich self-sustained dynamics, which is chaotic for some values of the control parameters ε_1, ε_2, α, β [28]. The following values for them are selected on this Letter: $\varepsilon_1=\varepsilon_2=1.0$, $\alpha=0.5$, $\beta=-1.75$. The system displays in that case chaotic behaviour with three different asymptotic attractors (Figure 1). We have concentrated on controlling the pair of nonsymmetric attractors, because their attraction basin is larger that for the symmetric one (Figure 2).

Two different forms of perturbing eqs. (4) have been tried, namely

$$\beta \rightarrow \beta + \delta\beta\left(\frac{x-\bar{x}}{p}\right) \qquad (5)$$

$$\varepsilon_1 \rightarrow \varepsilon_1 + \delta\varepsilon_1\left(\frac{y-\bar{y}}{q}\right) \qquad (6),$$

$\delta\beta$, $\bar{x}$, p, $\delta\varepsilon_1$, $\bar{y}$, q constants. Both of them have been found to be capable of controlling the chaotic system for rather small values of $\delta\beta$, $\delta\varepsilon_1$, ranging from 4% to 8% of the nominal values of β and ε_1. Let us consider (5) for definiteness. $\bar{x}$, p, $\bar{y}$, q are usually selected by computing the mean and maximun swing of x and y prior to switching the control on. In that way $(x-\bar{x})/p$ and $(y-\bar{y})/q$ are oscillating signals of zero mean and maximum peak-to-peak amplitude equal to one. In general, however, $\overline{x}$, p, $\overline{y}$, q, are to be considered as new adjustable (control) parameters. Figures 3 and 4 show the dynamics obtained for two different values of $\delta\beta$ (0.08 and 0.16) and it is compared with the signal before control was established. Locking to previously unstable periodic trajectories is clearly seen. A more detailed evaluation of the range on which an affective control can be achieved is obtained by computing a bifurcation diagram with $\delta\beta$ as the varying parameter (Figure 5).

In Figure 6 we present a first-return map approach to the dynamics of eqs. (4). It is represented the value of the (n+1)-esim relative maximum as a function of the n-esim relative maximum. In the chaotic state we obtain a cloud of scattered points distributed in a more or less parabolic shape. Embedded or very near to these points we have the discrete ones corresponding to the stabilized period-4 and period-2 orbits, which is a further evidence that we have really stabilized previously existing periodic trajectories, and not created them *ex novo* .

As a final test we have computed the Fourier spectrum for both chaotic (prior to control) and periodic signals, and, as shown in Figure 7, the sharp peaks expected for periodic signals in Figs. 3 and 4 coincide rather well with some peak-like features of the chaotic signal. These peaks are reminiscences of the periodic orbits embedded in the chaotic attractor.

It is interesting to note that the other non-symmetric attractor can be also controlled by the same kind of perturbation, just by changing the sign of $\delta\beta$.

In summary, whe have shown that it is possible to control a chaotic system by modulating a control parameter with a non-delayed part of an output signal of the system. Control is achieved for rather small values of the perturbation, and from the practical point of view its implementation is rather straighforward, as shown in Figure 8. The ultimate reasons that make this method work are thought to be twofold: on the one side the dimension of the control space is enlarged with the introduction of new adjustable parameters and on the other hand the overall structure of the vector field is smoothly and slightly changed. Due to the fact that a chaotic attractor contains a dense set of unstable periodic orbits, even these small changes are able to stabilize some of them. It is perhaps worth noting that the physical phase space of the system is not changed, i.e. its dimension is preserved, in contrast with other methods discussed in the literature which enlarge *simultaneously* the control parameter and phase spaces, sometimes even making this last one infinite-dimensional (control through delayed back-action).

For the particular model studied no new qualitative asymptotic behaviours are introduced when switching the control on, due to the small values of $\delta\beta$. Nevertheless for $\delta\beta/\beta \approx 1$ global changes on the dynamics should be expected, and in that case perhaps we should not talk about *control* anymore.

More than one asymptotic attractor can be controlled just by changing the sign of $\delta\beta$ and it is thus expected that the proposed method can be applied to more complex dynamical systems on which coexistence of attractors, or even of spatio-temporal structures are commonplace.

The author would like to acknowledge the inspiration brought by some recent work on dynamical control of chaos while developing these ideas, specially Refs. [24-27]. It is hoped that the proposed method could be soon tested on real physical systems and see if it actually provides a tool for helping controlling chaos.

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Figure Captions

Fig. 1. (a) x-z plane projection of the asymptotic attractors of eqs. (4). Solid line represents the inversion-symmetric attractor while the two non-symmetric ones are drawn with dashed lines. (b) Time histories for signals in the non-symmetrix attractor (solid line) and in the symmetric one (dashed line).

Fig. 2. Attraction basins for the attractors of Fig. 1 , computed for initial conditions of the form $(x_i,0,z_i,0)$ with $x_i \in [-5,5]$ and $z_i \in [-5,5]$. Deep and light blue corresponds to each of the nonsymmetric attractors, and green to the symmetric one.

Fig. 3. (a) Strange attractor (dashed line) and period four trajectory embedded on it and made stable for $\delta\beta = 0.08$ (thick solid line). (b) Time histories corresponding to the orbits in (a).

Fig. 4. (a) Strange attractor (dashed line) and period two trajectory embedded on it and made stable for $\delta\beta = 0.16$ (thick solid line). (b) Time histories corresponding to the orbits in (a).

Fig. 5. Bifurcation diagram with $\delta\beta$ as the control parameter. Regions of periodic motion are clearly seen for absolute values of $\delta\beta/\beta$ ranging from 4% to 14%.

Fig. 6. First-return map approximation to the dynamics on the nonsymmetric attractor, and the closed trajectories corresponding to period four (filled circles) and period two (filled rombs), obtained after control.

Fig. 7. (a) Fourier spectra for the chaotic trajectory (solid line) and the period four orbit (dashed line). (b) The same as (a) but for the period two orbit. Note than in both cases the sharp peaks

in the spectra of periodic signals coincide remarkably well whith more wide peak-like features in the chaotic spectrum.

Fig. 8. Block diagram for the experimental implementation of the proposed method.

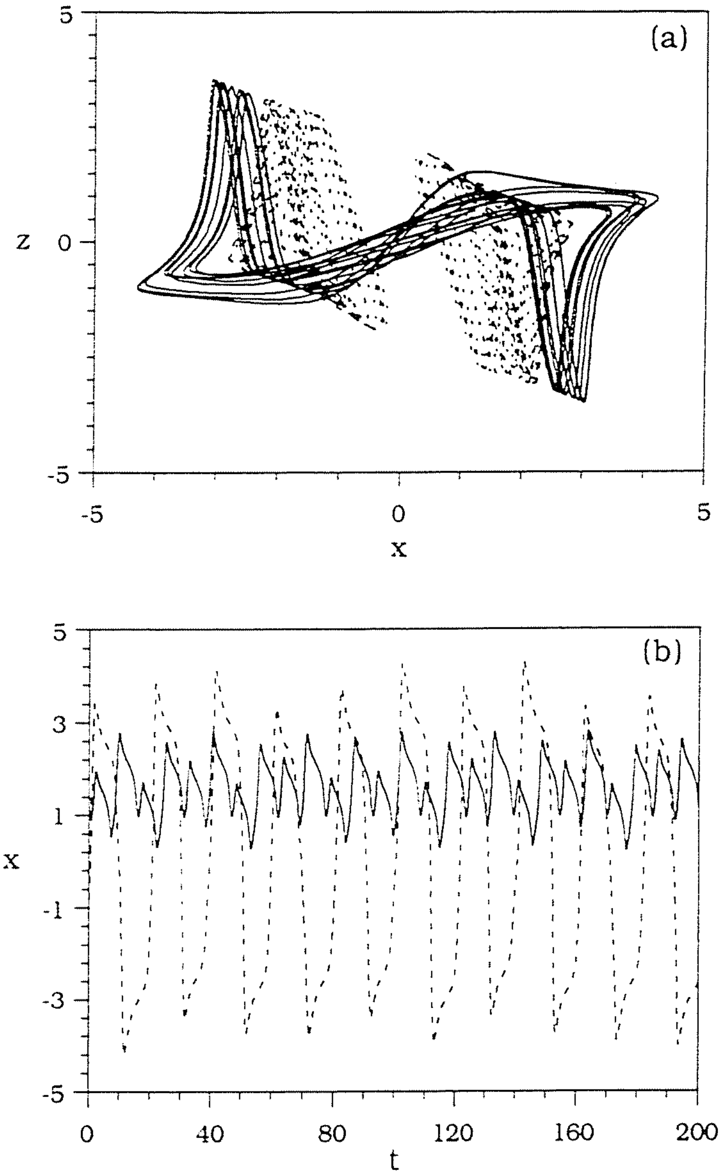


FIGURE 1

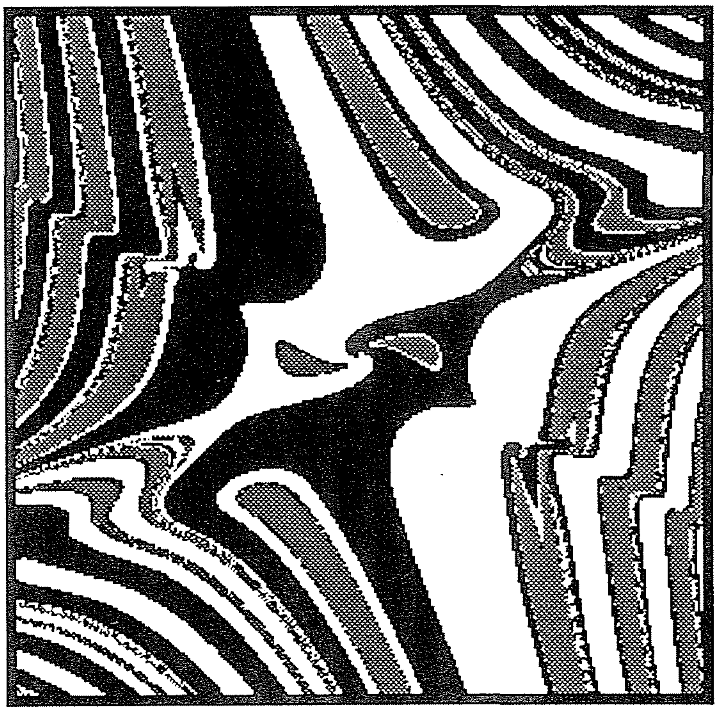


FIGURE 2

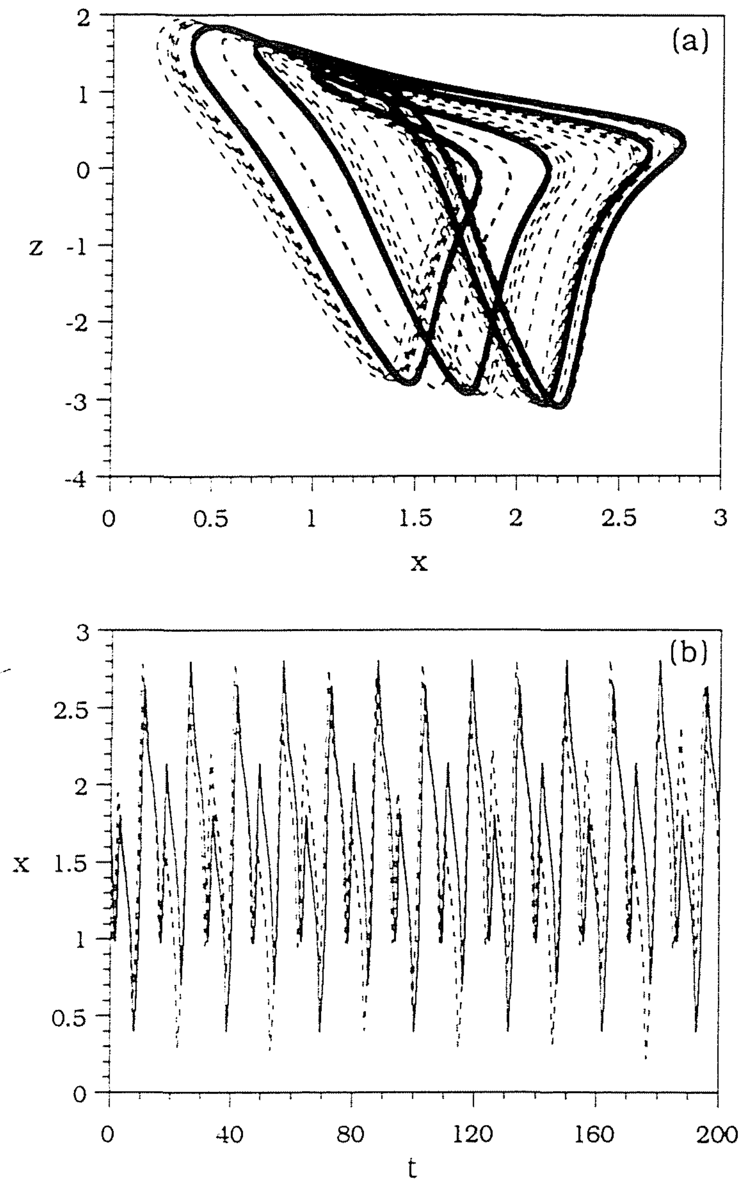


FIGURE 3

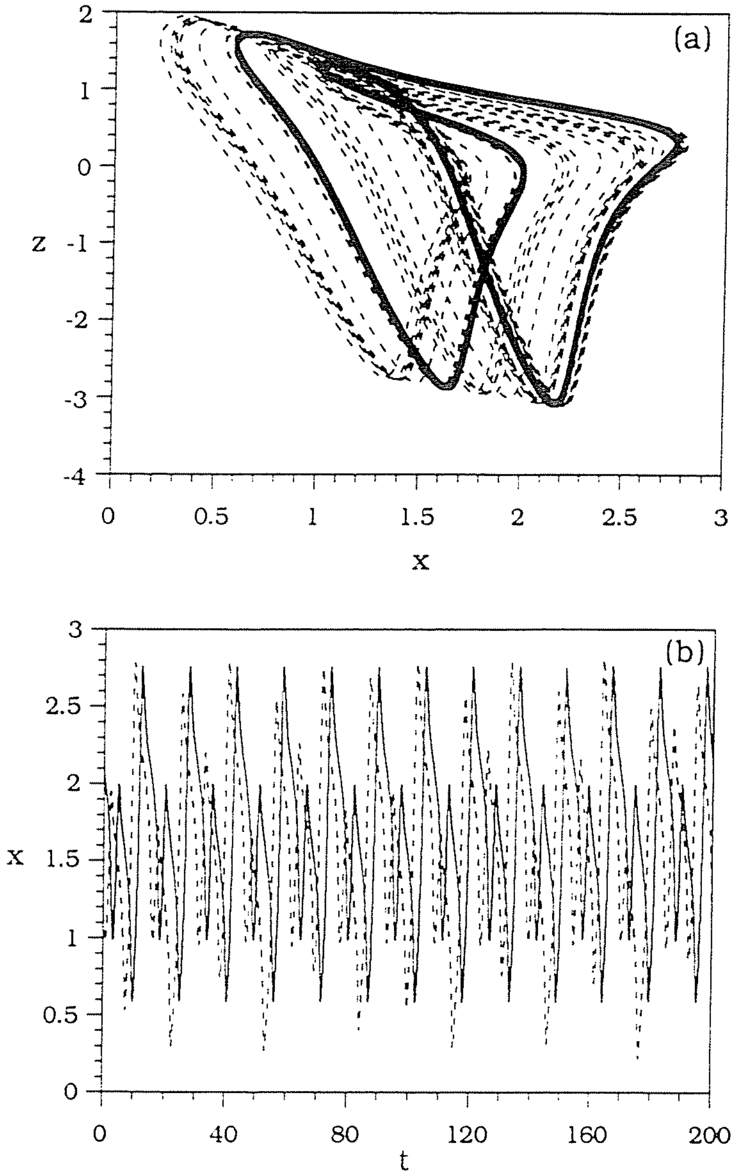
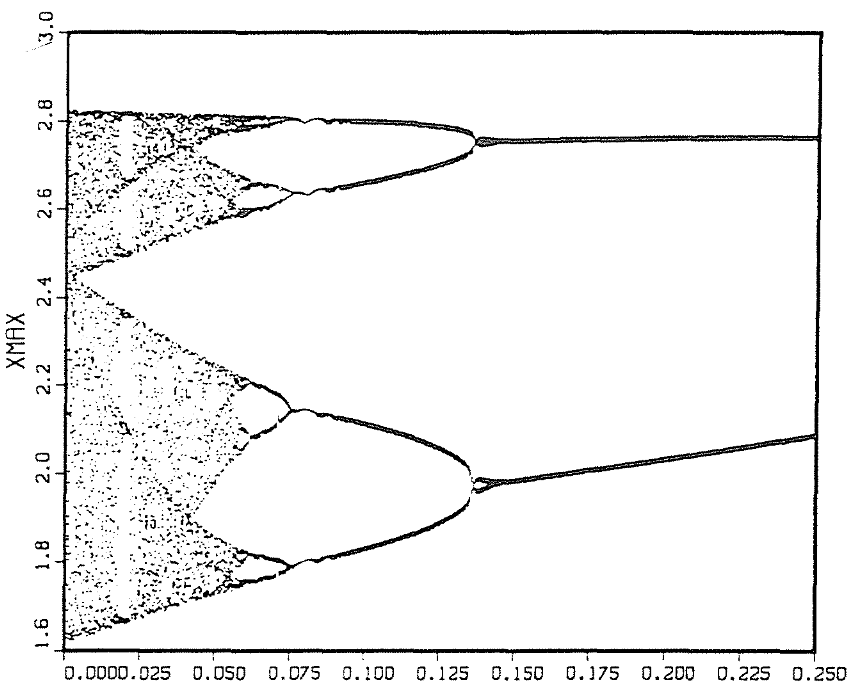


FIGURE 4



XMAX
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
0.0000.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
0.225
0.250

FIGURE 5

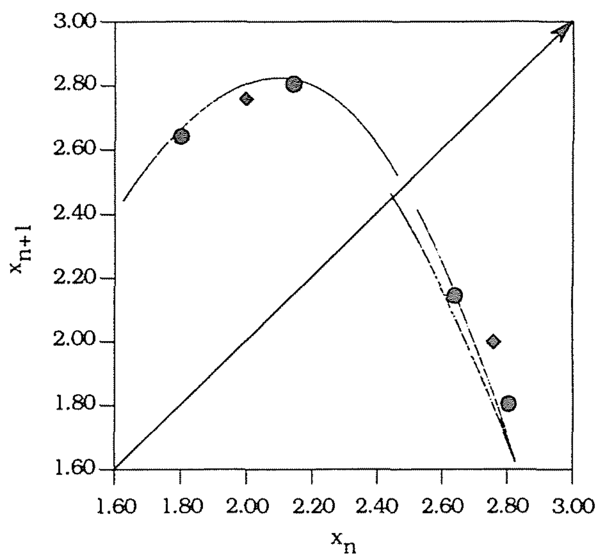

3.00
2.80
2.60
2.40
2.20
2.00
1.80
1.60
x_{n+1}
1.60 1.80 2.00 2.20 2.40 2.60 2.80 3.00
x_n

FIGURE 6

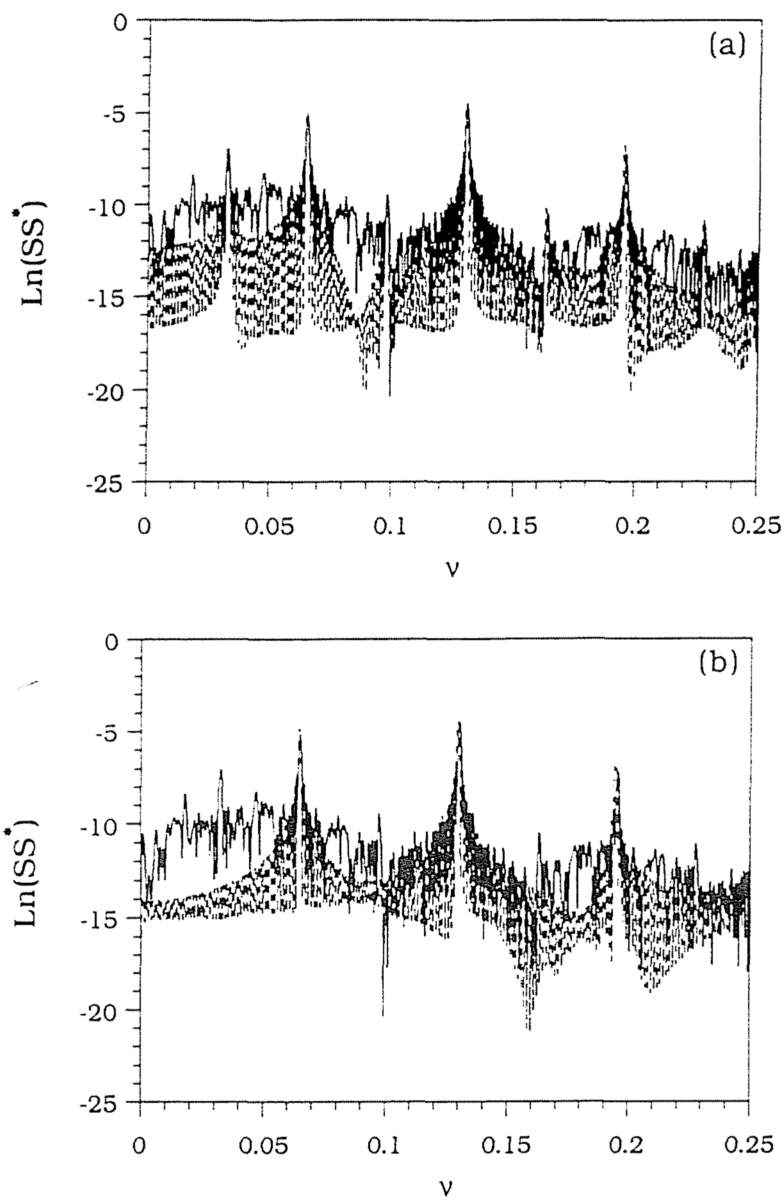
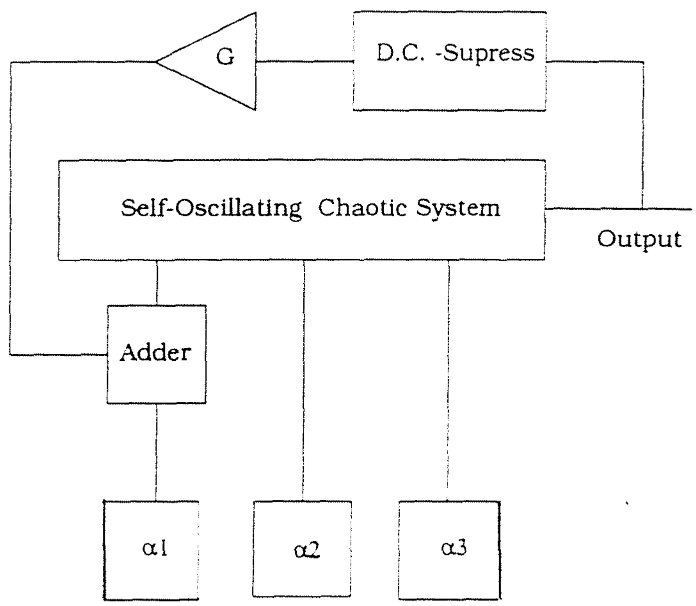


FIGURE 7



G
D.C. -Supress
Self-Oscillating Chaotic System
Output
Adder
α1
α2
α3

FIGURE 8

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"Control de un sistema caótico mediante modulación de un parámetro de control."

PASTOR, I. (1994) 28 pp., 8 Fisgs., 28 Refs.

Se propone un método para conseguir el control activo de un sistema caótico basado en modular un parámetro de control con una pequeña parte de la señal de salida del propio sistema. Desde un punto de vista teórico, este método es capaz de estabilizar el caos porque se permiten pequeñas modificaciones del campo vectorial que controla la dinámica, lo que a su vez hace que algunas de las soluciones periódicas inestables previamente existentes en el atractor extraño se estabilicen. Este método se inspira en trabajos muy recientes relacionados con estos mismos problemas de control en sistemas no lineales. Una de sus características más atractivas es que es muy sencillo llevarlo a la práctica en experimentos concretos. El método se ha ensayado en un sistema de ecuaciones diferenciales ordinarias que modelan el acoplo de dos circuítos electrónicos autooscilantes (osciladores de van der Pol). Se comenta la posibilidad de aplicarlo a sistemas espaciotemporales complejos en los que pueden coexistir múltiples estructuras caóticas para ciertos valores de los parámetros de control.

CLASIFICACION DOE Y DESCRIPTORES: 990200. Differential Equations. Control Theory. Control Systems. Feedback. Vector Fields.

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"Controlling a Chaotic System through Control Parameter Self-Modulation."

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A method for obtaining active control of a chaotic system based on the modulation of a control parameter by adding to it a small perturbation proportional to one output signal is proposed. From a theoretical point of view, chaos can be stabilized in the framework of this method because small modifications of the vector field controlling the dynamics are allowed, and thus some of the previously existing unstable periodic trajectories can be made stable. The method is much inspired on recent treatments of some related problems, and it is compared with them. One of its most attractive features is that it should be very easy to implement it on real experiments. The method is tested on a system of ordinary differential equations modelling the coupling of two self-oscillating electronic circuits (van der Pol oscillators). Some brief comments are made on the possibility that it could be applied to complex spatio-temporal systems where multiple chaotic structures can coexist for some values of the control parameters.

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