Turbulence and Electric Field Measurements by Doppler Reflectometry in TJ-II

Master Thesis presented by

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Abstract

Turbulence wave-number spectrum has been measured using Doppler reflectometry for ECH deuterium plasmas at the TJ-II stellarator. Plasmas with densities below and above the critical one, at which the so-called low-density transition occurs, have been studied. The possible impact of the ion mass on turbulence is addressed by the comparison of the present results with others previously obtained in hydrogen plasmas.
Chapter 1

Introduction

1.1 World’s Energetic Situation

Finding never-ending, cheap and non-contaminant energetic resources dates back to the industrial revolution. Nevertheless, up to date, we have already used up most of the limited energy resources discovered over history. This situation has become so critical in the past years that we must look forward to the future in order to stop a greater energetic crisis.

It is widely known that the world population will have increased to about 9000 million people in the next four decades; this will mean an increase of over 30% of the current world population. The improvements in the living conditions by then would also entail an increment of the double or triple of the present energetic needs, and at the moment, a unique technology capable of supplying for these requirements does not exist. On the other hand, we need efficient and non-contaminant energy sources in order to reduce the effects of the unpredictable consequences of climate change. In view of this situation, which is turning into one of the most important and challenging difficulties faced by the Earth, effective and accurate solutions are required. In this sense, thanks to the outstanding research in Energy carried out by scientists all over the world, an answer to the needs of our society will soon be discovered.

Several types of sources of energy are under research, others already developed and some others are being already improved. Among them, we could stand out the three energy sources, which will be decisive for our immediate future.

The first one to be stand out is the Solar Energy. This type of energy will be for sure decisive for our future. However, in spite of the obvious advantages, its efficiency, unfortunately, is still very far away from being able to take care of the worldwide energetic needs and is feasible in some countries. In any case, a lot more research is needed for this solar energy to be efficient.

Another option that will be important is Nuclear Fission, which constitutes a way of obtaining energy free of carbon emissions to the atmosphere, but it is not well accepted in our society due to the its radioactive wastes.

Finally, an experimental source of energy, which is able to combine effi-
ciency and cleanliness, is Nuclear Fusion. It does not need much space unlike solar or wind energy, it uses an almost inexhaustible source of resources, it is safe, it does not leave long-life radioactive wastes unlike Nuclear Fission and does not pollute the atmosphere. The only trouble that Nuclear Fusion faces is its technological complexity, but the results obtained over the tireless years of research foresee a success in the near future.

1.2 Nuclear Fusion

A process by which two light nuclei are combined to form a heavier one is called nuclear fusion, and it is followed by a great release of energy. For this reaction to happen, the primary nuclei have to approach each other enough to be able to interact. The ease of this process vanishes when the Coulom barrier between those primary nuclei plays its role, i.e. we need great energy to get these two nuclei close enough due to Coulomb repulsion [1]. Hence, in order to overcome the Coulombian repulsion and get high enough number of reactions, high temperatures of the order of the 10-25 KeV and densities of the order of \(10^{20} - 10^{22} \, \text{m}^{-3}\) must to be achieved.

Some nuclear reactions have been studied over the years. However, some of them are more promising than others. The most important fusion reactions are shown below:

\[
\begin{align*}
1H^1 + 1H^1 & \rightarrow 1D^2 + e^+ + \nu \quad (1.44 \, \text{MeV}) \quad (1.1) \\
1D^2 + 1H^1 & \rightarrow 2He^3 + \gamma \quad (5.49 \, \text{MeV}) \quad (1.2) \\
2He^3 + 2He^3 & \rightarrow 2He^4 + e^+ + 2\, 1H^1 \quad (12.86 \, \text{MeV}) \quad (1.3) \\
1D^2 + 1T^3 & \rightarrow 2He^4 + \, 0n^1 \quad (17.6 \, \text{MeV}) \quad (1.4)
\end{align*}
\]

where \(e^+, \nu\) and \(\gamma\) denote positron, neutrino and \(\gamma\)-ray respectively.

Nevertheless, from the previous nuclear fusion reactions, only reaction 1.4 is considered to be energetically favourable [1].

Obtaining deuterium and tritium is not a complicated process. Deuterium is easily found in nature (it can be extracted from seawater). Nonetheless, tritium is radioactive with a short life-time, which is not possible to find in nature. Luckily, tritium can be produced by means of bombarding lithium with neutrons from the deuterium-tritium fusion reactions by means of the following reactions:

\[
\begin{align*}
Li^6 + \, 0n^1 & \rightarrow 2He^4 + 1T^3 \quad (4.8 \, \text{MeV}) \quad (1.5) \\
Li^7 + \, 0n^1 & \rightarrow 2He^4 + 1T^3 + \, 0n^1 \quad (-2.47 \, \text{MeV}) \quad (1.6)
\end{align*}
\]

Hence, out of several possible cycles, the most attractive cycle of reactions is the D-T-Li [2]:
\[ ^1D^2 + ^1T^3 \rightarrow ^2He^4 + ^0n^1 \quad (17.6 \text{ MeV}) \quad (1.7) \]
\[ ^{Li^6} + ^0n^1 \rightarrow ^2He^4 + ^1T^3 \quad (4.8 \text{ MeV}) \quad (1.8) \]
\[ ^{Li^7} + ^0n^1 \rightarrow ^2He^4 + ^1T^3 + ^0n^1 \quad (-2.47 \text{ MeV}) \quad (1.9) \]

Any cycle considered has to work to a minimum temperature for the radiation losses to be compensated for the generated energy in the fusion process. In this sense, the deuterium-tritium cycle is the cycle that has smallest critical temperature (of about 5 KeV), being 15 KeV its optimum working temperature.

### 1.3 Ignition and Lawson Criterion

Nuclear fusion is not a trivial process. First, a vacuum chamber is needed in order to keep the future plasma isolated from the atmosphere and will also keep the plasma from impurities. Second, we need a certain configuration of magnetic fields, which will not only act as a precursor for the fusion to happen but also as a cage for the plasma not to collide with the walls. Third, once we are able to inject the primary nuclei in the machine, an ionization procedure carried out by heating the primary nuclei by means of external processes takes place. Once this is achieved, we need to get to a self-sustainable plasma, this means that the alpha particles produced in the fusion reactions mentioned earlier are able to sustain the temperature in steady state with no necessity of feed the system by external heating. This self-sustainable situation is called Ignition and the condition for which this occurs is called Lawson Criterion.

Let’s consider a confined plasma of density \( n \) to a temperature \( T \). Its thermal energy, \( 3nT \), decreases with time due to heat conduction and particle losses. Denoting as \( P_C \) to the confined energy rate loss, and \( \tau_E \) to the energy confinement time, it is defined [3]:

\[ \tau_E = \frac{3nT}{P_C} \quad (1.10) \]

which is commonly used in fusion science as a measure of plasma confinement. Lawson established that, for a confined plasma to produce enough fusion energy being able to compensate the necessary electric power to maintain the discharge, it is necessary that [4]:

\[ P_C + P_r = \eta(P_F + P_C + P_r) \quad (1.11) \]

where \( P_r \) is the radiation power losses, \( P_F \) the fusion power generated, and \( \eta \) is the conversion from thermal energy to electrical energy efficiency.
The l.h.s of equation 1.11 represents the power losses in the plasma, which is compensated by means of electrical power generated, represented by the r.h.s of the equation. The condition given by equation 1.8 may be expressed in a more convenient manner:

\[ n \tau_E \geq f(T) \]  

(1.12)

where \( f(T) \) is a function of the temperature which, for the deuterium-tritium reaction and a value for efficiency of about 1/3, is of the order of \( 3 \times 10^{19} \, m^{-3}\)s to the optimum temperature. For other cycles of reactions, the value to achieve is even higher.

Nevertheless, this condition is not enough. A fusion reactor must work in ignition regime, in which the alpha particles produced by the fusion reactor stay in the plasma heating it, compensating the radiation losses with no need of exterior energy contribution. Denoting \( P_\alpha \) to the alpha heating power, the ignition condition may be written as [5]:

\[ P_C + P_r = P_\alpha \]  

(1.13)

This condition is equivalent to the Lawson criterion for \( \eta = 0.136 \). On the other hand, for the cycle of deuterium-tritium fusion reactions, in terms of the \( n \tau_E \) product, the ignition condition can be rewritten as it follows [3]:

\[ n \tau_E \geq 1.5 \times 10^{20} \, m^{-3}s \]  

(1.14)

to a temperature of 30 KeV. Hence, in order to reach the ignition condition, it is necessary to confine a plasma of high enough density and temperature during a long enough period of time.

### 1.4 Magnetic Configurations

Some mechanisms to isolate the plasma from the atmosphere have been proposed and used over the years, but the leading ones are the toroidal magnetic configurations. Among them, we will emphasize two of them, tokamaks and stellarators, shown in figures 1.1 and 1.2 respectively, being these the leading ones in the scientific fusion community. The way of confining plasma in a torus-like machine requires a toroidal and a poloidal magnetic field, which, as a result, generate a helical magnetic field that acts as a "cage" that holds and shapes the plasma (and keeps it away from the machine walls).

In tokamaks, the poloidal magnetic field is generated by an intense toroidal current induced in the plasma as a result of the current flowing in the primary coil (transformer). Therefore, tokamaks work in pulse form, being this one its worst disadvantage. Furthermore, the great currents that flow in the plasma (of the order of MA) can produce violent instabilities, resulting in disruptions [3], which terminate the plasma with a huge loss of heat and particles over
the reactor walls, with consequent deterioration. On the other hand, tokamaks are the most advanced configurations in which most promising results have been obtained, having overcome the Lawson criterion [8]. The current biggest tokamak in operation and the one which has obtained the best results is JET (Joint European Torus), based at Culham, England. Currently, ITER (International Thermonuclear Experimental Reactor) is being under construction. It is pretended, once the ignition condition is reached by this reactor, to show the world the feasibility of energy generation by means of thermonuclear fusion.
All configuration in which the poloidal magnetic field is produced by a series of helicoidal coils placed along the torus is called Stellarator [9]. Therefore, stellarators are purely three-dimensional configurations. Furthermore, since all currents flow along external coils, it can work in steady state, being this one its principal advantage. Moreover, they are also more versatile, because external parameters can be adjusted. Nonetheless, the fact of being three-dimensional brings along their principal disadvantage (i.e. their complexity). This three-dimensionality can be extended to its theoretical study.

1.5 Motivation of the thesis

There is evidence that deuterium discharges have improved confinement properties compared with hydrogen ones. This improvement in the confinement properties attributed to the ion isotopic mass is called isotopic effect and has received interest because it could be beneficial for the D-T operation in ITER [10].

Collected results show that an isotopic effect exists [11], however its not yet understood. The step size of collisional transport increase as the ion mass increases, and drift-wave turbulence creates transport through flow cells, which increases as the normalized gyro-radius (which can be translated into mass) increases. Consequently, deterioration in the energy confinement time would be expected. Nonetheless, an enhancement of the energy confinement time is found.

Recent studies suggest that zonal flow dependence with the ion mass could explain isotopic effect [12]. Zonal flows (ZF) are stationary, poloidally and toroidally homogeneous and radially varying $E \times B$ flows created by nonlinear energy transfer from drift waves [13]. Hence, their generation reduces the intensity and level of transport caused by the drift wave turbulence.

Studies performed in TJ-II for hydrogen plasmas show that when a certain density is reached, the radial electric field is inverted near the plasma boundary [14]. Furthermore, when that threshold density is approached, the level of edge turbulent transport significantly increases in the plasma edge. This increment in the turbulence is attenuated once the threshold density is reached. Furthermore, there is also evidence that, at the transition in which the electric field is inverted, ZFs are created [15], which could be explained as a consequence of the previous increment of turbulence. In addition, a recent study [16] has shown that, when the plasma density approaches the critical density from below, the neoclassical viscosity drops, bringing along a low damping rate of the radial electric field, which could explain the generation of ZFs near the transition. The generation of these ZFs may be the responsible for the reduction of the turbulence once the threshold density is reached.

A campaign of deuterium as working gas was initiated in TJ-II in April 2012. This chance was taken to perform measurements on density turbulence with the purpose to study the influence of the ZF over turbulence in deuterium
plasmas for later comparison with results obtained in hydrogen plasmas.

This master’s thesis presents a study of density turbulence in D plasmas by means of Doppler reflectometry (a diagnostic technique that allows measurements of density fluctuations and its perpendicular rotation velocity), and is organized as follows: Chapter 2 gives an introduction to turbulence followed by the fundamental theory of microwave reflectometry. In chapter 3, the Heliac flexible TJ-II is introduced and a later description of the Doppler reflectometer used for the measurements is presented. Chapter 4 shows the experimental results; firstly, a short description of the experiment and the methods for data analysis is given. Later, measurements of the perpendicular rotation velocity (and its dependence with the turbulence scale) and perpendicular wave-number spectra of density turbulence are presented. Further comparison with the results obtained in H plasmas reported in [17] is performed. Finally, chapter 5 contains the conclusions of this work.
1. Introduction
Chapter 2

Theory

This chapter will be dedicated to the theoretical part of the master’s thesis. Firstly, the basic principles of turbulence will be described. Secondly, theoretical foundations of microwave reflectometry will be explained focusing in Doppler reflectometry. Thirdly, relevant aspects such as microwave propagation and reflection in plasma will be discussed in detail. Finally, a description of the Doppler reflectometer system installed on TJ-II stellarator is also included.

2.1 Turbulence

Turbulence can be defined as a chaotic or unstable eddying motion in a fluid, gas or plasma. The importance of its study becomes evident in physics since turbulent motion is the natural state of most fluids [18].

Fluids are in continuous interaction with boundaries and obstacles, which break their laminar structure into eddies. In the case of fusion plasmas, the plasma itself is also subjected to external forces due to gradients produced when it is magnetically confined, conferring it a turbulent behavior.

The most basic equation to describe turbulence in incompressible neutral fluids is the Navier-Stokes equation,

\[ \rho_m \frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\nabla p + \nu \Delta u, \]

(2.1)

Where \( \rho_m \) is the mass density, \( u \) is the velocity field, \( p \) the pressure and \( \nu \) the viscosity.

Equation 2.1 can be rewritten in dimensionless magnitudes defining \( x' = x/L, \quad u' = u/U_0, \quad t' = t/T \) and \( p' = p/\rho U^2 \), being \( L \) the characteristic scale of the system, \( U_0 \) its characteristic velocity and \( T \) a characteristic time. Furthermore, the spatial and temporal derivatives are of the form \( \nabla' = L \nabla \) and \( \partial/\partial t' = (L/U_0) \partial/\partial t \). Substituting these changes in equations 2.1 we reach the dimensionless form of the Navier-Stokes equation:
\[ \rho_m \left( \frac{\partial w}{\partial t} + (w \cdot \nabla') w \right) = -\nabla' p' + \frac{1}{R_e} \nabla' \omega, \]  
(2.2)

Where the Reynolds number, \( R_e \), which describes the degree of turbulence of the system, is defined as follows:

\[ R_e = \frac{U_0 L \rho_m}{\nu} \]  
(2.3)

where \( U_0 \) is the characteristic velocity of the fluid, \( L \) its characteristic length, \( \rho_m \) the density of the fluid, and \( \nu \) its kinematic viscosity.

As the value of this number increases, the formation of eddies structures is more pronounced.

In fusion plasmas, at least two fluids (electrons and ions) exist in continuous interaction with each other, resulting in a much more complex system to describe. The previous explanation gives just a basic understanding of how turbulence is created.

### 2.1.1 Spectral Properties of Turbulence

Turbulence is a very complex phenomenon to describe. Due to its complexity and the number of variables involved, an analytical solution is not viable. On the other hand, a statistical description to study turbulence and try to make predictions on its behavior can be used.

An important matter when studying a turbulent system is how much turbulence energy is contained in each scale. The processes by which the turbulence energy is transferred between scales are called cascades, where the cascade towards smaller scales is termed direct cascade and the cascade towards larger scales inverse cascade. Immediately, the question of how this energy is transferred between different scales emerges, which answer depends on whether the turbulence considered is two or three-dimensional.

The theory given by Kolmogorov in 1941 (K41-Theory) describes how the energy is transferred between different scales in homogeneous and isotropic 3D turbulence. This theory is based on a dimensional analysis, which assumes that energy is injected at some large scales to be subsequently transferred to smaller scales (inertial range), in which the energy is non-linearly transmitted by a continuously decreasing sequence of eddies and controlled by an energy transfer rate. Finally, the remaining energy is dissipated through viscosity (dissipative range). Figure 2.1 illustrates the latter process. For self-similar Navier-Stokes turbulence, the K41-Theory predicts the spectral energy per unit wave-number as follows:

\[ E_k = C \epsilon^{2/3} k^{-5/3} \]  
(2.4)
2.1 Turbulence

where $C$ is the Kolmogorov constant and $\epsilon$ is the energy transfer rate. The spectral index predicted by K41-Theory is $\alpha = -5/3$.

In contrast, the effect of vorticity stretching present in 3D turbulence is absent in the 2D turbulence, resulting in the conservation of enstrophy. The principal consequence of this vorticity absence is that the energy is transferred...
towards larger scales while enstrophy towards smaller scales, each with different spectral indexes.

The theoretical wave-number spectrum for 2D Navier-Stokes turbulence is shown in figure 2.2. In this scheme, if the energy is injected at one scale $k_i$, two inertial ranges emerge. Energy is then transferred from the injection scale towards larger scales and enstrophy is transferred from the injection scale to smaller scales. These cascades are commonly labelled inverse cascade and direct cascade respectively, and so, the term dual cascade is often used when referring to 2D turbulence. At small scales, the energy is dissipated and consequently contributes to the total energy in the system. Regarding the spectral indexes, the inverse cascade is characterized by $\alpha = -5/3$ while the direct cascade shows $\alpha = -3$ [19].

The description given above provides us with a basic understanding of how turbulence develops. Nonetheless, this description only holds when a neutral fluid is considered. In magnetized plasmas, at least two fluids (electrons and ions) are in continuous interaction with each other, resulting in a much more complex system to describe.

A property of magnetically confined plasmas in toroidal devices is the generation of nested flux surfaces, produced by the magnetic field $\mathbf{B}$. This magnetic field, as previously mentioned, can be separated in its intrinsic components, a toroidal component $B_\phi$, and a poloidal component $B_\theta$, where $\phi$ and $\theta$ are the toroidal and poloidal angles respectively. In stellarators, both $B_\phi$ and $B_\theta$ are obtained through a complex arrangement of magnetic coils.

In a plasma in force equilibrium (i.e. $\nabla p = j \times \mathbf{B}$, where $p$ is the pressure, $j$ the diamagnetic current and $\mathbf{B}$ the magnetic field) [3], plasma parameters on the magnetic flux surfaces are nearly constant since particles can move freely along the magnetic field lines, therefore parallel transport is fast. In contrast, perpendicular to the flux surfaces, the transport is slow due to the Lorentz force, and so gradients (density, temperature, etc.) are formed. Hence, turbulence properties for this system is very different from the one mentioned above for a neutral fluid.

### 2.2 Microwave Reflectometry

Microwave reflectometry is used in fusion plasmas in order to measure the electron density profile, plasma instabilities, density turbulence, etc. There are two types of reflectometry, conventional reflectometry and Doppler reflectometry. In conventional reflectometry, a probing beam is launched into the plasma perpendicularly to the cut-off layer (i.e. $\theta = 0^\circ$ with respect to the cut-off layer normal), where is reflected back, and detected. Doppler reflectometry makes use of a tilt angle between the probing beam and the cut-off layer normal to measure the Bragg back-scattered signal.
2.2 Microwave Reflectometry

2.2.1 Electromagnetic Wave Theory

Considering a travelling plane wave moving in the $x$ direction as follows:

$$A(x, t) = A_0 \exp[i(kx - \omega t)] \quad (2.5)$$

where $A_0$ is the amplitude, $k = 2\pi/\lambda$ is the wave-number and $\omega = 2\pi f$ the angular frequency of the wave. We can define the phase velocity of the wave as the propagation rate of a point of constant phase.

$$v_{ph} = \frac{dx}{dt} = \frac{\omega}{k} \quad (2.6)$$

If we now consider a superimposition of two sinusoidal waves with slightly different frequencies and wavelength, we can define the group velocity as the velocity at which the variations in the shape of the amplitude (i.e. modulation) propagates in space, and is given by:

$$v_{gr} = \frac{\partial \omega}{\partial k} \quad (2.7)$$

The relation between the angular frequency $\omega$ and the wave number $k$ is known as dispersion relation. All the information about the propagation of a wave, such as $v_{ph}$, $v_{gr}$, reflection points, etc., is contained in this relation. If $\omega(k)$ is directly proportional to $k$ (as it occurs in the case of ion acoustic waves), then the group and phase velocities coincide. If not, the envelope of the wave will deform as it propagates. This phenomenon is know as dispersion.

An additional fundamental property of an electromagnetic wave, which the dispersion relation does not contain, is its polarization. An electromagnetic wave has both an oscillating electric and magnetic component perpendicular to each other. The orientation and phase of the electric field vector of a wave defines the polarization state.

Microwave Propagation in Plasmas

Since microwaves have electromagnetic nature and travel at the speed of light, $c$, their propagation in plasma is governed by Maxwell’s equations. The two relevant Maxwell’s equations for our next study are the following:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.8)$$

$$c^2 \nabla \times \mathbf{B} = \frac{j}{\varepsilon_0} + \frac{\partial \mathbf{E}}{\partial t} \quad (2.9)$$
2. Theory

where \( \mathbf{E} \) is the electric field, \( \mathbf{B} \) the magnetic field, \( \mathbf{j} \) the vector current density, and \( \varepsilon_0 \) the permittivity of free space [3]. The time derivative of equation 2.9 and the curl of equation 2.8 are respectively:

\[
c^2 \nabla \times \frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\varepsilon_0} \mathbf{j} + \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{2.10}
\]

\[
\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla \times \frac{\partial \mathbf{B}}{\partial t} \tag{2.11}
\]

Now, assuming an electromagnetic wave of the form \( E \approx \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)] \) and combining equations 2.10 and 2.11, we get to the linear wave equation

\[
\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \frac{i\omega}{\varepsilon_0 c^2} \mathbf{j} + \frac{\omega^2}{c^2} \mathbf{E} \tag{2.12}
\]

Moreover, since microwaves have high frequencies, ions can be considered as fixed so the current \( \mathbf{j} \) comes from electron motions:

\[
\mathbf{j} = -n_0(x) e \mathbf{v}_e \tag{2.13}
\]

where \( \mathbf{v}_e \) is the electron velocity, which can be obtained from the equation of motion

\[
\frac{\partial \mathbf{v}_e}{\partial t} = -\frac{e}{m} (\mathbf{E} + \mathbf{v}_e \times \mathbf{B}_0) \tag{2.14}
\]

where \( \mathbf{B}_0 \) is the plasma magnetic field. Solving equation 2.12 in components leads to [20]:

\[
(k_y^2 - k_0^2 + \kappa^2) E_x + i k_y \frac{\partial E_y}{\partial x} - \frac{i \omega c}{\omega} \kappa^2 E_y = 0 \tag{2.15}
\]

\[
 i k_y \frac{\partial E_x}{\partial x} + \frac{i \omega c}{\omega} \kappa^2 E_x - \frac{\partial^2 E_y}{\partial x^2} + (\kappa^2 - k_0^2) E_y = 0 \tag{2.16}
\]

\[
\frac{\partial^2 E_z}{\partial x^2} + k_0^2 (1 - \frac{\omega_p^2}{\omega^2} - \frac{k_y^2}{k_0^2}) E_z = 0 \tag{2.17}
\]

where

\[
k_0 = \omega/c \tag{2.18}
\]

\[
\kappa^2(x) = k_0^2 \frac{\omega_p^2}{\omega^2 - \omega_e^2} \tag{2.19}
\]

\[
\omega_p^2 = \frac{n_0 e^2}{\varepsilon_0 m} \tag{2.20}
\]
$\omega_c = \frac{eB_0}{m}$ (2.21)

$\omega_p$ and $\omega_c$ are defined as the plasma frequency and electron cyclotron frequency respectively [20]. Equations 2.15 to 2.17 are for the case of a wave entering the plasma obliquely in the plane perpendicular to $B_0$ ($k_z = 0$). Since in Doppler reflectometry the incident microwave is launched into the plasma with a tilt angle, this case will be described.

**The Ordinary Mode**

From the above set of equations, equation 2.17 constitutes the wave equation for the ordinary mode at oblique incidence. Here, the electromagnetic wave propagates at a non-zero tilt angle $\theta_0$ in the $x$ direction. The radial electric field $E$ is parallel to the magnetic field $B_0$ so that $E = E_z \hat{z}$ and $B_0 = B_0 \hat{z}$. From Snell’s law, $k_y = k \sin \theta$ is constant and so $k_y = k_0 \sin \theta_0$. Equation 2.17 can then be re-written as:

$$\frac{\partial^2 E_z}{\partial x^2} + k_0^2 \left( 1 - \frac{\omega_p^2}{\omega^2} - \sin^2 \theta_0 \right) E_z = 0$$ (2.22)

Where the index of refraction, $\epsilon$ can be expressed as:

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} - \sin^2 \theta_0$$ (2.23)

A cut-off occurs when $\epsilon$ is equal to zero, whereas resonance occurs when $\epsilon$ is equal to infinity. As a wave propagates in a plasma in regions where $\omega_p$ and $\omega_c$ are changing, it will encounter resonances and cut-offs. Typically, a wave is reflected at a cut-off and absorbed at a resonance. Consequently, setting equation 2.14 to zero and solving for $\omega$, one obtains the condition for a cut-off in ordinary mode:

$$\omega = \omega_p \cos \theta_0$$ (2.24)

For normal incidence, the $\sin^2 \theta$ term in equation 2.22 vanishes and cut-off occurs at $\omega_p = \omega$. This means that for oblique incidence, cut-off occurs at a lower value than $\omega_p$.

**The Extraordinary Mode**

In the ordinary mode, the electron motion is unaffected by $B_0$. When $E$ is perpendicular to $B_0$, the electric field has both an $x$ and $y$ components ($E = E_x \hat{x} + E_y \hat{y}$). Thus, the magnetic field affects the electron motion. A wave with $E \perp B$ is an extraordinary wave and is governed by equations
2. Theory

2.15 and 2.16. These equations can be solved for $E_y$ by eliminating $E_x$ and simplified by means of a parametrization factor $s = k_0x$ [20].

$$\frac{d^2E_y}{ds^2} + p(s)\frac{dE_y}{ds} + q(s)E_y = 0$$

where

$$p(s) = \frac{d}{ds}(\omega_p)^2 \frac{\sin^2\theta_0}{(\omega^2 - \omega_h^2)(\cos^2\theta_0 - \frac{\omega^2}{\omega^2 - \omega_h^2})}$$

$$q(s) = 1 - \frac{\omega_p^2 \omega^2 - \omega_p^2}{\omega^2 \omega^2 - \omega_h^2} - \sin^2\theta_0 - \frac{d}{ds}(\omega_p)^2 \frac{\omega_c}{\omega} \frac{\sin\theta_0 \cos^2\theta_0}{(\omega^2 - \omega_h^2)(\cos^2\theta_0 - \frac{\omega^2}{\omega^2 - \omega_h^2})}$$

Hence, the index of refraction is now:

$$\epsilon = 1 - \frac{\omega_p^2 \omega^2 - \omega_p^2}{\omega^2 \omega^2 - \omega_h^2} - \sin^2\theta_0$$

Finally, setting equation 2.28 to zero, one obtains two solutions for cut-off in extraordinary mode:

$$\omega_R = \frac{1}{2} \left( \omega_c + \sqrt{\omega_c^2 + 4 \frac{\omega_p^2}{\cos\theta}} \right)$$

$$\omega_L = \frac{1}{2} \left( -\omega_c + \sqrt{\omega_c^2 + 4 \frac{\omega_p^2}{\cos\theta}} \right)$$

$\omega_R$ and $\omega_L$ are known as the right-hand and left-hand cut-offs respectively. It is noted that $\omega_R$ is greater or equal to the plasma frequency, $\omega_p$. The extraordinary mode is generally used to study edge plasmas because it is not possible to measure at low frequencies at the edge (ordinary mode).

2.2.2 Doppler Reflectometry

In Doppler reflectometry, a microwave beam is launched from a transmitting antenna into the plasma at a non-zero angle $\theta_0$ with respect to the normal to the cut-off layer. The incident beam is reflected and Bragg scattered at a cut-off layer in the plasma.

A first approach of Doppler reflectometry can be explained by considering a reflection grating containing a small sinusoidal corrugation and moving in vacuum [21] as figure 2.3 shows. This grating can be used as a simple model of density perturbations in the plasma.

The back-scattered signal is then detected by the launching antenna. As a result, the Bragg scattering equation gives the condition
2.2 Microwave Reflectometry

Figure 2.3: Principle of Doppler reflectometry illustrated by a moving reflection grating in vacuum and a single antenna arrangement [21].

\[ k_\perp = 2k_0 \sin \theta_0 \]  

Where \( k_0 = 2\pi/\lambda_0 \) is the probing wave-number. Equation 2.30 shows that, by varying the tilt angle, the Doppler Reflectometer selects plasma density perturbations with finite wave-numbers \( k_\perp \) in the cut-off layer by the Bragg condition. Therefore, the diagnostic provides access to the \( k \)-spectrum of the turbulence.

Now, if the plasma fluctuations move with a velocity \( V_\perp \), a Doppler shift \( \omega_D \) is detected in the received microwave signal. It is given by

\[ \omega_D = \mathbf{V} \cdot \mathbf{k} = V_\perp k_\perp + V_\parallel k_\parallel + V_r k_r \]  

In magnetically confined plasmas, by means of aligning the launching wave perpendicular to the magnetic field \( \mathbf{B} \), Doppler Reflectometers are only sensitive to \( k_\perp \) and not to \( k_\parallel \). Furthermore, it is usually assumed that \( k_\perp \gg k_\parallel \) and \( V_\perp < V_\parallel \). In addition, turbulence does not displace itself in the radial direction, and therefore, the second term in equation 2.32 is dropped, leading to:
\[ \omega_D = 2\pi f_D = V_\perp k_\perp \quad (2.33) \]

Where the perpendicular wave-number of density turbulence \( k_\perp = 2k_{\perp,i} \), and

\[ f_D = V_\perp \left( \frac{2}{\lambda_0} \right) \sin\theta_0 \quad (2.34) \]

Equation 2.30 is defined considering a plasma slab. Nevertheless, in magnetically confined plasmas, the cut-off layer curvature has to be taken into account. Therefore, the perpendicular wave-number of density fluctuations is determined by ray or beam tracing codes, which gave the wave-number of the incident ray \( k_{\perp,i} \) and the radial position of back-scattering.

To summarize, Doppler reflectometry makes use of a tilt angle between the probing beam and the cut-off layer normal, to measure the Bragg back-scattered beam. This technique allows measurements of turbulence and its perpendicular rotation velocity at different turbulent scales [21].
Chapter 3
Experimental Set up

This chapter will focus on an introduction to the Heliac Flexible TJ-II and its main characteristics followed by a short description of the Doppler reflectometer.

3.1 Heliac Flexible TJ-II

Starting its operation in 1998, the TJ-II is a 4-period heliac-type stellarator located at "Centro de Investigaciones Energéticas, Medioambientales y Tecnológicas" (CIEMAT), in Madrid (Spain). It was the result of combined efforts between the Fusion Division at CIEMAT in collaboration with the "Oak Ridge National laboratory" (ORLN) in the United States and the "Institut fur Plasmaphysik" (IPP) in Garching, Germany. A schematic representation of the TJ-II stellarator is shown in figure 3.1. A set of 32 toroidal magnetic field coils create the magnetic field shape to confine the plasma, and the 4 vertical field coils control the plasma position. Finally, two central coils, one of them circular and the other helical, define the 3-D twist of the machine.

The combined action of the magnetic fields generated by the coils create a bean-shaped magnetic surface as can be seen in figure 3.2, which guide the plasma particles around the torus. Moreover, due to the 4-periodicity symmetry, the heliac flexible TJ-II is divided in 4 sectors, which have an equivalent flux surface for each 90°. Table 3.1 shows its operational parameters.

Conveniently, a great versatility enables the TJ-II stellarator to vary its rotational transform $\iota$, which is defined as the number of poloidal rotations in one toroidal rotation, and can be expressed as follows:

$$\iota = \frac{R_0 B_\theta}{\rho B_\phi} \quad (3.1)$$

where $B_\phi$ and $B_\theta$ are the toroidal and poloidal magnetic fields, respectively, $R_0$ the major radius, and $\rho$ the effective radius.

This variation of $\iota$ can be done just by varying the ratio between the current flowing in the HX (Helicoidal coil) and the current flowing in the CC (Central
3. Experimental Set up

![Schematic representation of the Heliac Flexible TJ-II](image)

Figure 3.1: Schematic representation of the Heliac Flexible TJ-II [7].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major Radius ($R_0$)</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Average Minor Plasma Radius $\langle a \rangle$</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Average Magnetic Field (B)</td>
<td>1T</td>
</tr>
<tr>
<td>Toroidal Field Coil Number</td>
<td>32</td>
</tr>
<tr>
<td>Rotational Transform $\iota(0)$</td>
<td>0.9-2.2</td>
</tr>
<tr>
<td>Electron Density ECRH</td>
<td>$1.7 \times 10^{19} \text{ m}^{-3}$</td>
</tr>
<tr>
<td>Density NBI</td>
<td>$8 \times 10^{19} \text{ m}^{-3}$</td>
</tr>
<tr>
<td>Maximum Electron Temperature</td>
<td>2 keV</td>
</tr>
<tr>
<td>Ion Temperature</td>
<td>150 eV</td>
</tr>
</tbody>
</table>

Table 3.1: Technical data and typical plasma parameters of the Heliac Flexible TJ-II.

coil). In this sense, $\iota$ can be varied from 0.9 to 2.2.
3.2 The Doppler Reflectometer

In this section, a short description of the Doppler reflectometer installed in TJ-II is given. For details of the Doppler reflectometer, the reader is referred to [22].

Doppler Reflectometer System is adapted to the three-dimensional geometry of the experiment. Due to the curvature of the plasma, careful attention was paid in the design and fabrication of the two main parts that compose the Doppler Reflectometer (i.e. the antenna and the ellipsoidal mirror).

The antenna was developed to achieve a Gaussian beam in the Q-band \( f = 33 - 50 \text{ GHz} \). The emitted beam has a beam waist of \( 1.7\lambda_0 \) at 41 GHz and the beam waist scales linearly with the square root of the wavelength (i.e. \( \omega \propto \sqrt{\lambda_0} \)). This property results in constant radius of curvature of the incoming beam. The ellipsoidal mirror, is steerable, allowing to change the launch angle of the beam with respect to the perpendicular incidence by \( \pm 20^\circ \) with an angular resolution of \( \pm 0.1^\circ \). This angular variation enables a perpendicular wave-number measurements in the range of \( 3 - 15 \text{ cm}^{-1} \). Thanks to the steerable mirror, the reflectometer can work both as a conventional reflectometer (i.e. \( \theta_0 = 0^\circ \)) and as a Doppler reflectometer (oblique incidence). The value of the perpendicular wave-number is calculated after each discharge using the 3D ray-tracing code TRUBA with input density profiles. Figure 3.3 shows the results of TRUBA for different frequencies and different angles of incidence.

To gather the measurements, the Doppler Reflectometer has two independent channels, in which the frequencies during a discharge can be either constant or changed independently (in a range of 33-50 GHz) on a time-scale smaller than 1 ms.

Figure 3.4 shows the circuit diagram of one channel. Two independent synthesizers of a frequency range of \( f = 8 - 12.5 \text{ GHz} \) are used [23], called...
main synthesizer (RF) and local oscillator (LO). The latter has a frequency offset of 0.192 GHz.

The RF signal is multiplied by 4 to achieve $33 - 50 \text{ GHz}$ and later transmitted by the antenna to the plasma. The reflected signal is sent to a harmonic mixer at an intermediate frequency of $768 \text{ MHz}$ and amplified. Finally, the signal is sent to the I.Q. detector.

The LO signal is mixed with RF signal and then multiplied by 4 to achieve $768 \text{ MHz}$ and then sent to the I.Q. detector from which the information of plasma parameters is registered.
Figure 3.4: Circuit diagram of one channel of the fast frequency hopping reflectometer. The signal received from the plasma is down converted to 768 MHz and heterodyne detection, done by an I.Q. detector, yields the complex amplitude signal which contains the information on the plasma properties [23].
Chapter 4

Experimental Results

This chapter contains the experimental results. Firstly, a short description of the analysis methods is given. Later, measurements of the perpendicular rotation velocity of density fluctuations and its dependence with the turbulence scale are presented. Finally, the perpendicular wave-number spectra of density turbulence is studied.

The discharges used in this work correspond to ECH heated deuterium plasmas. The proportion of deuterium in the plasma was of about 50%. The reflectometer was programmed to do a frequency swept from 33 to 42 GHz (channel 1) and from 33.5 to 42.5 GHz (channel 2) with frequency steps of 1 GHz for each discharge. Also, the operational angle of the mirror was varied between discharges from 32.5° to 39°. This allows measurements of perpendicular rotational velocity of density turbulence and density fluctuations as later described. The magnetic configuration was 101_42_64 for all discharges, and the heating power was \( P_{ECRH} = 500 \text{ kW} \).

4.1 Data Analysis

This section describes the analysis carried out in this master’s thesis. First, an introduction to the quadrature phase detection is given followed by the process used to obtain the Doppler shift \( f_D \).

4.1.1 Quadrature Phase Detection

The heterodyne detection technique allows to measure the phase and the amplitude of the reflected microwave. The detected beam has two signals, the in-phase \( I \) and quadrature \( Q \) components:

\[
I = A \cos \phi \quad (4.1)
\]

\[
Q = A \sin \phi \quad (4.2)
\]
which are combined to obtain the complex amplitude of the signal as follows:

\[ V = I + iQ = Acos\phi + iAsin\phi = Ae^{i\phi} \]  \hspace{1cm} (4.3)

Since \( V \) is a complex number, the fluctuation spectra are two-sided (i.e. positive and negative frequencies are resolved). This allows to know in which direction the phase is evolving, which, in case of Doppler reflectometry means the direction of propagation of density fluctuations (i.e. the sign of \( V_\perp \)).

### 4.1.2 Numerical Methods Used for Data Analysis

The data analysis of this work was done by means of IDL (Interactive Data Language) programming language. The signal received is firstly time windowed (i.e. 0.2 ms per window) and a fast Fourier transform is applied to the complex amplitude signal \( C = I + iQ = Ae^{i\phi} \). This gives a double-sided power spectrum \( S(f) \).

In order to show the measurement process, we have selected three representative discharges. Figure 4.1 shows a typical time windowed spectrogram for low density. The frequency swept of the reflectometer is of about 30 ms and its repeated during the whole discharge. The white stripes present in the spectrum are due to the frequency change. The color map reflects the amplitude of the Doppler peak which is proportional to the density fluctuation level. For low density the Doppler peak is found to be at negative frequencies as well as for near to critical densities. For high density, on the contrary, the Doppler peak is found to be at positive frequencies. As shown in chapter 2, the frequency of the Doppler peak permits the calculation of the perpendicular rotation velocity of density turbulence.

![Figure 4.1: 30221. CH2. Time windowed spectrogram for low density.](image)

Figure 4.2 shows the power spectrum for the same discharge as in figure 4.1 measured at a frequency of 35.5 \( GHz \). By means of a Gaussian fitting, we are
4.2 Perpendicular Rotation Velocity of Density Fluctuations

Figure 4.2: Power spectrum of complex signal, using sliding FFT analysis technique for the same discharge than in figure 4.1 at 35.5 GHz

able to know the values of the amplitude of the Doppler peak, its frequency and the width of the Gaussian fit.

Finally, the Doppler shift $f_D$ is directly related to the perpendicular rotation velocity of density fluctuations as follows

$$V_\perp = -2\pi \left( \frac{f_D}{2k_i} \right)$$

(4.4)

where $k_i$ is the wave-number of the incident ray, calculated by means of a ray tracing code. Note also that the sign of $f_D$ reveals the direction of the plasma rotation. If $f_D < 0$ (i.e. low density), then $V_\perp$, and the plasma rotates in the ion-diamagnetic direction; if $f_D > 0$ (i.e. high density), then $V_\perp$, and the plasma rotates in the electron-diamagnetic direction.

4.2 Perpendicular Rotation Velocity of Density Fluctuations

Since turbulence theory predicts a possible tearing apart of turbulent structures when the velocity shear is sufficiently large [24], correct measurement and analysis of $V_\perp$ may help to understand turbulence suppression mechanisms.

Investigations performed in TJ-II have shown that ECRH plasmas change their rotation from ion-diamagnetic to electron-diamagnetic direction, when plasma density exceeds a threshold value, which depends on ECH power level and on magnetic configuration, [25].
Figure 4.3 shows the line-averaged density $n_e$ (black solid line) modulated in time, and radial electric field (in red) measured at $\rho = 0.8$. It is observed that, at low densities, the radial electric field is positive and so it is $V_\perp$, revealing the ion-diamagnetic direction of plasma rotation in TJ-II. On the other hand, for high density, the radial electric field is negative and so it is $V_\perp$, revealing the electron-diamagnetic direction of plasma rotation.

Doppler Reflectometry allows to measure the perpendicular rotation velocity, $V_\perp$, which has two contributions as follows

$$V_\perp = v_{E \times B} + v_{ph}$$

(4.5)

where $v_{E \times B} = E_r/B$ (being $E_r$ the radial electric field) and $v_{ph}$ the mean phase velocity of density fluctuations. This latter velocity is small in some experiments compared to $v_{E \times B}$ [21].

In this section, by means of the Doppler Reflectometer installed in TJ-II, $V_\perp$ profiles were obtained. Positive and negative $V_\perp$ profiles in ECRH deuterium plasmas were observed for different plasma conditions.

Figure 4.4 shows $V_\perp$ for three selected discharges (30221, 30230 and 30250). For a density of $0.5 \times 10^{19} \text{ m}^{-3}$ (lower than the threshold density), as expected, we find the $V_\perp$ to be positive in the whole radial range (rotation in the ion-diamagnetic direction), where a velocity of about 2 km/s is found close to the
plasma edge. A small dip is found at $\rho = 0.85$ followed by an increase in velocity up to about 4.5 km/s as we get further inside the plasma. On the other hand, for a density of $0.8 \times 10^{19} m^{-3}$, we find the perpendicular velocity to be negative in the whole radial range. The velocities in the high density case vary from about -1.5 km/s to -2.5 km/s close to the plasma edge and slows down to the value to about -0.7 km/s for a radial position of $\rho = 0.68$ (deepest position reached).

For a density of $0.6 \times 10^{19} m^{-3}$, close to the critical density, a similar behavior than in the case of low density is exhibited. A velocity of about 1.5 km/s was found close to the plasma edge (at about $\rho = 0.9$), then a decrease is found as we move inwards, finding a velocity of about 1 km/s at $\rho = 0.85$, finally, as we get further inside the plasma, we obtain velocities of up to 3 km/s.

Qualitatively, the behavior of $V_\perp$ is the same as the one found in hydrogen plasmas (reported in [17]) even though it was not possible to get closer to the
critical density.

Figure 4.4 also shows the errors bars of the data. It can be seen that the errors in the radial position at which the cut-off layer is located increase as the probing frequency increases. This agrees with the fact that, as the center of the plasma is approached, first, the curvature of the cut-off layer is more pronounced, and second, the density gradient decreases as we get further inside the plasma, resulting on greater errors. The errors in the value of the perpendicular velocity are computed taking into account the wave-number, its uncertainty and the perpendicular velocity. It is found that the errors vary from about 0.1 to 0.2 km/s in the low density case, and up to 0.1 km/s in the case of high density. The same behavior was found in the work performed for hydrogen plasmas in [17].

4.3 Velocity Dependence with the Turbulence Scale

This section gives a study of the $V_\perp$ with the perpendicular wave-number. This is motivated by the fact that, since $v_{E\times B}$ does not depend on the wave-number and, if $v_{ph}$ is small compared with $v_{E\times B}$, a calculation of the radial electric field would be feasible.

In order to minimize the dependence of $V_\perp$ with the effective radius, the data considered for this section are those measured in a small interval $\Delta \rho = 0.05$. This allows us to have access to the dispersion relation. Figure 4.5 shows the evolution of the mean perpendicular rotation velocity of density fluctuations with the perpendicular wave-number for low (in blue) and near-to-critical density (in yellow). A clear dependence of $V_\perp$ with $k_\perp$ can be seen.

For both line-averaged densities, $V_\perp$ exhibits a variation of a factor 2 when $k_\perp$ varies between $3 cm^{-1}$ and $10 cm^{-1}$ (which can be translated into a behavior of the phase velocity of density fluctuations, since the $v_{E\times B}$ does not depend on the perpendicular wave-number).

Since it was not possible to penetrate so deep into the plasma (i.e. $\rho = 0.6 \pm 0.05$) for high density, a similar study is carried out at $\rho = 0.8 \pm 0.05$. Figure 4.6 shows the evolution of the mean perpendicular velocity of density fluctuations with the perpendicular wave-number, for high density, at $\rho = 0.8 \pm 0.05$. Once again, a variation of a factor 2 is exhibited by $V_\perp$ for the same perpendicular wave-number interval than in figure 4.5.

Hence, we can conclude that a dependence of the phase velocity $v_{ph}$ with the turbulence scale has been found (i.e. $\omega_{flux}(k_\perp) = k_\perp v_{flux} \propto k_\perp^\alpha$ with $\alpha \neq 1$).

The obtained dependence of $V_\perp$ with the turbulence scale entails high errors in the calculation of radial electric field. Figure 4.5 shows, in yellow points, the wave-number dependence of the perpendicular rotation velocity of density fluctuations for a density of $0.6 \times 10^{19} m^{-3}$. The perpendicular rotation velocity of density fluctuations is varying from 2 km/s to nearly 4 km/s. Since we know
4.3 Velocity Dependence with the Turbulence Scale

Figure 4.5: Evolution of the mean perpendicular velocity of density fluctuations (in the ion diamagnetic direction) vs the perpendicular wave-number at $\rho = 0.6 \pm 0.05$ for low density (in blue) and for near to critical density (in yellow).

Figure 4.6: Evolution of the mean perpendicular velocity of density fluctuations (in the electron diamagnetic direction) vs the perpendicular wave-number at $\rho = 0.8 \pm 0.05$. 
that the $v_{E \times B}$ does not depend on the turbulence scale, it is $v_{ph}$ the one giving that dependency with the perpendicular wave-number. Therefore, the error in the calculation of the radial electric field would be very high. Due to this reason, the radial electric field will not be calculated in this work in order to avoid uncertainties.

4.4 Perpendicular Wave-number Spectra of Density Turbulence

Apart from being one of the few quantities which allow detailed comparison between experiments and theory, the perpendicular wave-number spectrum gives information about the repartition of fluctuation energy over the different spatial scales. It also contains information about the character of underlying instabilities and the mechanisms involved in energy transfer between different scales.

In this section a study of the perpendicular wave-number spectra of density turbulence at the edge of the plasma for three line-averaged densities of $0.5 \times 10^{19}$, $0.6 \times 10^{19}$ and $0.8 \times 10^{19} m^{-3}$ is performed.

The $S(k_\perp)$ spectra for a radial interval of $\rho = 0.75 – 0.9$, are shown in figures 4.7, 4.8 and 4.9, for low, critical, and high density respectively.

It is a general observation that, for the same wave-number range, the density turbulence level decreases, towards smaller scales, nearly two orders of magnitude for the three line-averaged densities.

In order to characterize the perpendicular wave-number spectra, the normalized wave-number $k_\perp \rho_s$ is often used, where $\rho_s$ is the Larmor radius of the ions, defined as,

$$\rho_s = \frac{\sqrt{2mT_e}}{eB}$$  \hspace{1cm} (4.6)

where $m$ is the hydrogen/deuterium mass, $T_e$ is the electron temperature, $e$ is the electron charge, and $B$ is the magnetic field.

Moreover, the concentration of deuterium reached during the experiment was of about 50%, therefore, a separate calculation of $k_\perp \rho_s$ will be shown for hydrogen and deuterium, where the larmor radius for H and D have been found to be $\rho_s(H) = 0.25 \text{ cm}$ and $\rho_s(D) = 0.35 \text{ cm}$ respectively.

With regard to the structure of the wave-number spectra, firstly, for low density, as shown in figure 4.7, the spectrum in the lowest wave-number range is compatible with a power law $S(k) \propto k^{-\alpha}$ with spectral index of $\alpha \approx -1.40$, while this power law representation starts to break down around the normalized wave-numbers $(k_\perp \rho_s)_H = 1.35$ and $(k_\perp \rho_s)_H = 1.89$ (i.e. $k_\perp = 5.4 \text{ cm}^{-1}$) where the spectral power decreases faster ($\alpha \approx -5.44$). Secondly, as can be seen in figure 4.7, for near to critical density, the power law is characterized
4.4 Perpendicular Wave-number Spectra of Density Turbulence

Figure 4.7: Perpendicular Wave-number Spectrum for a line-averaged density of $0.5 \times 10^{19} \text{ m}^{-3}$ in the radial range $\rho = 0.75 - 0.9$. by an spectral index of $\alpha \approx -2.11$ and breaks down at normalized wave-numbers of $(k_{\perp} \rho_s)_H = 1.93$ and $(k_{\perp} \rho_s)_D = 2.70$ (i.e. $k_{\perp} \approx 7 \text{ cm}^{-1}$) where decreases towards higher wave-numbers ($\alpha \approx -6.54$). Finally, as figure 4.9 shows, for high density, the spectrum is characterized by a power law with an spectral index of $\alpha \approx -1.31$ at lower wave-numbers, breaks down at around $(k_{\perp} \rho_s)_H = 1.75$ and $(k_{\perp} \rho_s)_D = 2.45$ (i.e. $k_{\perp} = 7 \text{ cm}^{-1}$) and decreases as another power law with an spectral index of $\alpha \approx -5.60$.

The perpendicular wave-number spectra of density turbulence for low (in blue) and near-to-critical (in yellow) density is represented in figure 4.10. It can be seen that higher density fluctuations are measured at higher line-averaged density (i.e. near to critical density). A similar scenario is found in H plasmas [17], where for both low and near to critical line-averaged densities, the density fluctuations decrease was found to be of two orders of magnitude for the same radial interval considered in this work. Hence, we can conclude that, qualitatively, the level of fluctuations grow as we approach the threshold density in both hydrogen and deuterium plasmas.

The low purity of the plasmas analyzed in this work (i.e. a concentration of deuterium of about 50%) may be responsible for the lack of differences with
As studied in Chapter 2, this type of k-spectrum is not consistent with the theories of neutral fluid turbulence. Two-dimensional (2D) turbulence theory provides a mechanism of dual cascade. The energy cascade, characterized with a spectral index of $\alpha = -5/3$, and the enstrophy cascade, characterized with a spectral index of $\alpha = -3$. In addition, in 2D turbulence, the transition should be expected around the energy injection scale. On the other hand, in case of fusion plasmas, this idealized model is not to be expected since several instabilities may be present.

In general, through the perpendicular wave-number spectra of density turbulence measured, it is observed that the wave-number spectra are mainly composed of two regions compatible with a power law $S(k) \propto k^{-\alpha}$: at smaller wave-numbers, in which decays slower, and at high wave-numbers, in which decays faster. Also, from the comparison study, we can conclude that there are no big changes with respect to the work performed for H plasmas. In both
cases, the fluctuations grow as the critical density is approached.
Figure 4.10: Perpendicular Wave-number Spectrum for two line-averaged density of $0.5 \times 10^{19} m^{-3}$ (in blue) and $0.6 \times 10^{19} m^{-3}$ (in yellow).
Chapter 5

Conclusions

In the present master’s thesis, Doppler reflectometry was used in order to study the perpendicular wave-number spectrum and perpendicular rotation velocity of density turbulence of ECH deuterium plasmas for further comparison with previous results in hydrogen plasmas.

Firstly, the radial dependence of the perpendicular rotation velocity of density fluctuations $V_\perp$ was studied. Both ion-diamagnetic and electron-diamagnetic directions of the plasma were accessible in this experiment, therefore, positive and negative profiles of $V_\perp$ were observed. These profiles were found to exhibit a similar behavior to the ones in hydrogen plasmas.

Secondly, since Doppler reflectometry permits to access the $V_\perp$ dependence with the wave-number, which can then be used to obtain information about the dispersion relation of the turbulence measured inside the plasma, a further study of $V_\perp$ with the wave-number was performed. This latter study showed a clear dependence of perpendicular rotation velocity of density fluctuations with the turbulence scale. This dependence with the wave-number led to a very inaccurate calculation of the radial electric field. Therefore, in order to avoid high uncertainty in the calculations, the radial electric field was not computed in this work.

Finally, the wave-number spectrum gives the repartition of fluctuation energy over different spatial scales and gives detailed information about the character of underlying instabilities and to the mechanisms involved in energy transfer between different scales. Measurements showed that, for the same wave-number range, the density turbulence level decreases towards smaller scales of nearly two orders of magnitude for the three line-averaged densities studied. Moreover, the results showed that the wave-number spectrum is composed of two regions compatible with a power law $S(k) \propto k^{-\alpha}$: at smaller wave-numbers, a region in which the spectrum decreases slowly, and at larger wave-numbers, a region of energy transfer in which the spectrum decreases faster. Furthermore, the fluctuation level was found to grow as the critical density is approached. These results are very similar to the ones previously obtained in hydrogen plasmas.
In summary, no big changes have been found in the comparison of the fluctuation level of density turbulence between deuterium and hydrogen plasmas. This could be due to the fact that the proportion of deuterium in the plasma was of about 50%. Hence, more research on the impact of the ion mass on turbulence with a higher proportion of deuterium should be performed in the future for further comparison.
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5. Conclusions


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Declaration in lieu of oath

Herewith I declare in lieu of oath that I have prepared this thesis exclusively with the help of my scientific teachers and the means quoted by them.

Madrid, 08.10.2012

Aarón Hernández-Pérez

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