Brillouin limit for electron plasmas confined on magnetic surfaces

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As is well known, the density of pure electron plasmas that are confined by a magnetic field is limited by the Brillouin density, $n_B \equiv \epsilon_0 B^2/2m_e$. However, the electron density can be limited to a much lower value when the electrons are confined on magnetic surfaces, such as the surfaces produced by a stellarator. If the electron temperature is a spatial constant, the electron force-balance equation,

$$\frac{m_e}{e}\vec{v}\cdot\vec{\nabla}v + \frac{\vec{\nabla}p}{en} = \vec{\nabla}\phi - \vec{v}\times\vec{B},$$

can be rewritten as

$$\vec{\nabla}\phi_* = \vec{v}\times\vec{B}_*.$$

The effective electric potential and the effective magnetic field are

$$\phi_* \equiv \phi - \frac{m_e}{2e}v^2 - T\ln(n) \quad\text{and}\quad \vec{B}_* \equiv \vec{B} - \frac{m_e}{e}\vec{\nabla}\times\vec{v}.$$

The electron density for magnetic confinement in a cylinder with $\vec{B} = B\hat{z}$ is bounded by the Brillouin limit. If one assumes the electrons are pressureless and have a spatially constant density n_0, then $\phi = (en_0/4\epsilon_0)r^2$. Both $\vec{B}_*$ and ϕ_* vanish when $n_0 = n_B$, and the equation $\vec{\nabla}\phi_* = \vec{v}\times\vec{B}_*$ has no solutions for $n_0 > n_B$.

The confinement of electrons on magnetic surfaces is lost when the field lines of the effective magnetic field $\vec{B}_*$ leave the confinement region and strike the chamber walls. If the magnetic surfaces of the true field $\vec{B}$ are described by the toroidal flux, ψ_t that they enclose, so $\vec{B}\cdot\vec{\nabla}\psi_t = 0$, then confinement is easily lost when $\vec{\nabla}\psi_t\cdot\vec{\nabla}\times\vec{v}$ has Fourier terms that resonate with the rotational transform of $\vec{B}$, for then the $\vec{B}_*$ surfaces are split by islands. The resonant Fourier terms are given by $(\vec{B}\times\vec{\nabla}\psi_t)\cdot\vec{\nabla}(v_\parallel/B)$. In other words, resonant Fourier terms in the parallel flow of the electrons can cause a break up of the surfaces of the $\vec{B}_*$ field. The parallel flow is determined by the condition that $\vec{\nabla}\cdot(n\vec{v}) = 0$, which implies $\vec{B}\cdot\vec{\nabla}(v_\parallel/B) = -\vec{\nabla}\cdot(n\vec{v}_\perp)$. When $n_0 << n_B$, the divergence of the perpendicular flow is given by $\vec{\nabla}\cdot(n\vec{v}_\perp) = (\vec{B}\times\vec{\nabla}\phi)\cdot\vec{\nabla}(n/B^2)$. Variations in the geometry, which cause the electron density n to vary on the magnetic surfaces, and variations in the magnetic field strength on the magnetic surfaces can both drive resonant Fourier terms in $\vec{\nabla}\psi_t\cdot\vec{\nabla}\times\vec{v}$ that are proportional to n/n_B. These terms cause a loss of confinement when they are sufficiently large to destroy the surfaces of the effective magnetic field $\vec{B}_*$. This work was supported by the grant DE-FG02-95ER54333 from the U.S. Department of Energy and PHY-0317359 from the National Science Foundation.