COMMENTS

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Comment on "The Hurst exponent and long-time correlation" [Phys. Plasmas 7, 1181 (2000)]

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The paper "The Hurst exponent and long-time correlation" by G. Wang *et al.*[1] reaches a very surprising conclusion. The authors state: "It [the fact that H is well above 0.5] is found to be connected with short-time correlations.... These results suggest that the R/S method is not appropriate for evaluating long-time correlations in fusion devices." This conclusion is surprising in several respects. First, the R/S method[2] has been applied with success in many fields as a tool for investigating long-range dependencies. The relationship between a high value of the Hurst parameter, H, and self-affinity (i.e., long-term memory or long-range dependency) has been established in a large number of publications.[3] It is difficult to understand how and why the method should lead to different conclusions in the particular case of probe data from Tore Supra. Second, their conclusion does not answer the question of why and how short-range correlations should appear as long-range dependencies. Finally, the statement that this failure is generic to fusion devices is also surprising.

The test of the R/S method suggested by the authors is reasonable. It is an extension of the standard test of data shuffling, which we ourselves have used in testing our R/S algorithm.[4] The test consists of subdividing the data set into blocks of length N and shuffling the blocks. This should decorrelate any scales longer than N. One must, however, be careful when choosing a range of time lags over which the H exponent is determined. There is no discussion of this point in the paper, nor do the authors give any details about the ranges they have used. The range should be such that the R/S trace has a well-defined power scaling, it should include at least a decade of lag values, and its lower end should be at lags that are large compared to the size of the blocks N. Also, one should bear in mind that the high range of values of R/S has poor statistics (is calculated from few independent estimates), so that the accuracy of the R/S values decreases with increasing time lag. It is difficult to give a more detailed numerical prescription for the calculation of H. One should be guided by a careful examination of the data and the calculated R/S.

We have applied the test proposed by Wang *et al.* to a numerically generated time series of fractional Brownian motion[5] data with $H=0.8$. We have subdivided the sequence into blocks of 10, 100, and 1000 points and shuffled them following the prescription of Ref. 1. The results are shown in Fig. 1. For $N=10$, we can see a clear change in the slope of the R/S curve at about $\Delta T=100$. When we calculate H in the range $10^4>\Delta T>10^2$, where the R/S curve is close to a power law (a straight line in the double logarithmic plot), we obtain $H=0.52$. This is consistent with the absence of long-range dependencies, as expected. For $N=100$, we can see the change in slope at about $\Delta T=500$ and we estimate $H=0.50$ in the range $10^4>\Delta T>5\times10^2$. For $N=1000$, there are not enough points left on the sample to calculate H for larger lag values.

We have applied the same test to ion saturation current fluctuations from Langmuir probe measurements at the plasma edge in the Wendelstein VII Advanced Stellarator (W7-AS).[6] The data record used is longer (100.000 points) for improved clarity. The results are quite similar to the ones obtained for the fractional Brownian motion data. In Fig. 2(a) we show the R/S for these data for several of the applied tests. In this figure, we have only included the R/S values for $N=10$, 20, 100, 500, and 2000 to keep the picture clear. The effect of decorrelation induced by the shuffling of data blocks is obvious. There is a change in the slope of the R/S curve, separating two distinct scaling regions.

The results of Fig. 2(a) are represented schematically in Fig. 2(b). Clearly, the slope of the curves changes from its initial value ($H=0.8$) to its final value ($H=0.5$) at a point (the "break point") whose position depends on the value of the shuffling block length N. The position of this "break point" occurs at a lag value of about 5 times N. In all cases, the final slope is $H=0.5$, provided the position of the break point ($\approx 5N$) falls within the range of the graph, indicating that correlations for lags greater than $5N$ are destroyed. No

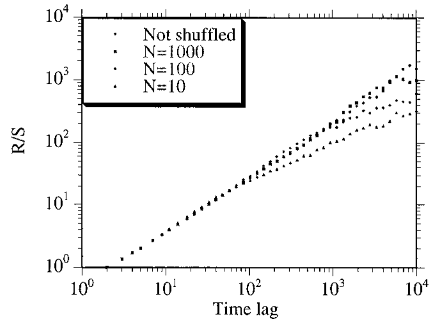

FIG. 1. R/S calculation for a fractional Brownian motion sequence with $H=0.8$. The sequence has been subdivided into blocks of size N that have been shuffled.

saturation effect in the sense of the comment of Wang *et al.*, is observed. If their conclusions were to hold, for values of N above a certain threshold the graph would no longer change. We see, however, that the position of the break point is directly proportional to N over the whole range of accessible values (which is quite a bit longer than the range available to Wang *et al.*). This fact that the position of the ''break point'' is proportional to N is a strong indication of the existence of long-range correlations and contradicts the conclusions by Wang *et al.* completely.

Evidently, the slope of the curve can only be determined by fitting a power law (i.e., a straight line on the double logarithmic plot) for those lag ranges within which the slope is constant. When the slope of the curve is determined from a lag range that incorporates the ''break point,'' the value of

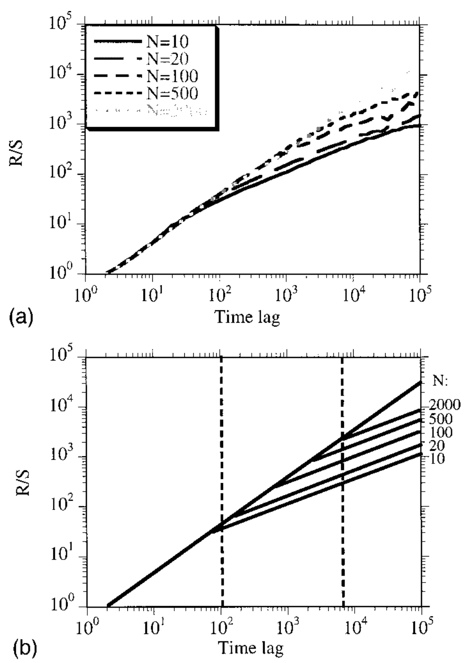

FIG. 2. R/S calculation for an ion saturation current time series from W7-AS: (a) results of the calculation breaking the sequence into blocks of size N that have been shuffled; (b) schematic representation of the same results.

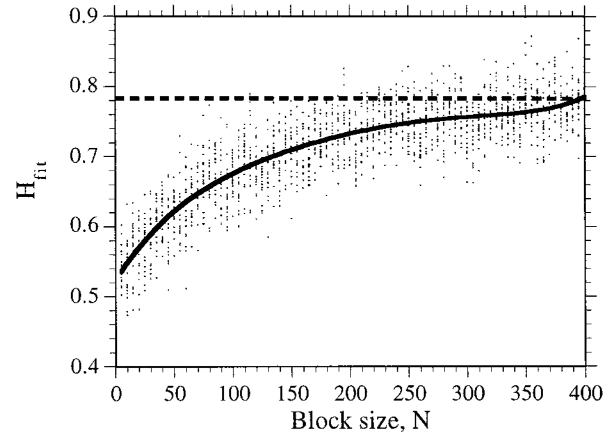

FIG. 3. Results from a blind fitting of the R/S with a power dependence for a fixed range of time lags and different N block sizes.

the ''slope'' that is determined is an average between the initial and final slopes, and is without physical meaning in itself.

This is, however, what Wang *et al.* appear to have done.[7] They have determined H from a fixed range of lags while varying N. The fixed lag range is indicated in Fig. 1(b) by the two vertical straight lines. When one attempts to determine H for each value of N by fitting a power law (i.e., a straight line on the double logarithmic plot) from the data within the fixed range indicated by the vertical lines in Fig. 2(b), one obtains Fig. 3. Because of the existence of a break point within the fit range, the obtained fitted value of H, $H_{\rm fit}$, is not equal to the value of either the first or the second slope (before and after the break point, respectively), but to an average of the two. Accordingly, this apparent H-value, $H_{\rm fit}$, varies slowly with the length of the shuffling data block, N, as indeed Wang *et al.*, present in Fig. 4 of their comment. But, again, this behavior only reflects the fact that the ''break point'' of the R/S curve moves through the fit range [cf. the vertical dotted lines in Fig. 2(b)] as N varies, and provides no information at all about the long-range correlations themselves.

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