

Magnetic equilibrium with fluid flow in a large $m=2$ island at RTP

B.Ph. van Milligen

(Asociación EURATOM-CIEMAT para Fusión, Avda. Complutense 22, Madrid, Spain)

N.J. Lopes Cardozo

(FOM-Institute "Rijnhuizen", PB 1207, 3430 BE Nieuwegein, The Netherlands)

Abstract

In recent work [1, 2] observations of a large $m=2$ island were presented that demonstrated the existence of a pressure gradient across the island. This fact disagrees with the usual assumption that the isobaric surfaces coincide with the flux surfaces. In [1, 2] a phenomenological model was proposed in which the observed gradients are explained from a short mean free path along the field lines due to the high collisionality in the outer regions of the plasma.

In this work we address the same problem from the equilibrium point of view. We describe the equilibrium in the presence of an island by means of a semi-analytical model. In the presence of pressure gradients along the flux surfaces the forces due to $\text{grad}(p)$ and $\mathbf{j} \times \mathbf{B}$ do not cancel at all points in space. Due to the pressure gradient along the field lines, particle transport will take place from the inside (high-pressure) region of the island to the outside (low-pressure) region. Preliminary results indicate that the pressure gradient across the field lines, along with the $\mathbf{j} \times \mathbf{B}$ force, can drive a compensating cross-field fluid flow from the outside to the inside. This dissipative flow only occurs at sufficiently low temperatures (high collisionality), because with low collisionality transport along the field lines is dominant.

We present a computer simulation of the $m=2$ island, partly based on experimental data and partly on assumptions. The simulation demonstrates that fluid flow may be important in equilibrium calculations in islands in the outer regions of the plasma.

1. Introduction

The computation of a tokamak equilibrium taking account of the existence of islands is a notoriously difficult problem, even when ideal MHD is assumed to be valid in most of the plasma volume. The situation becomes even more difficult when fluid flow must also be taken into account. Nevertheless, experimental observations (see section 2) lead us to believe that is exactly the situation we need to describe.

In this paper we make a first attempt at trying to understand the complex equilibrium with flow problem in magnetic islands. For that purpose, we rather arbitrarily define an "equilibrium" flux and afterwards check that the chosen flux functions indeed lead to a situation that can more or less be described as being in equilibrium. Of course, such a model cannot lay claim to a high degree of reality, but we believe the exercise is instructive. Then we observe how a deviation from equilibrium similar to the one we have observed in an experiment leads to fluid flows.

2. Experimental observations and interpretation

Previously, we have reported on the observation of a large $m=2$, $n=1$ island on RTP [1, 2]. The RTP tokamak has major radius $R_0 = 0.72$ m and the plasma minor radius is $a = 0.165$ m. The observations were made in an ohmically heated discharge with plasma current $I_p = 145$ kA, toroidal field $B_\phi = 2.1$ T, edge safety factor $q_a = 2.8$, central electron temperature $T_e(0) \approx 750$ eV and central electron density $n_e(0) \approx 5.5 \cdot 10^{19} \text{ m}^{-3}$. A large rotating $m=2$, $n=1$ mode occurred preceding a major disruption and lasted for about 12 ms., interrupted by a minor disruption 3 ms. before the major disruption. The analysis of the island, 2 ms. prior to the minor disruption, was made based on measurements of the poloidal field with a set of pick-up coils, a 20 channel heterodyne ECE radiometer and a 19 channel FIR interferometer. We observed both T_e - and n_e -gradients across the island. The T_e gradients could be understood by carefully analysing local transport across and along the field lines within the island, taking account of the mode geometry. The n_e gradients were more difficult to explain, although a possible cause for both types of gradients was found in mode rotation with respect to the plasma fluid. It was concluded that the island did not conform to the usual assumption that pressure is constant on a flux surface.

3. A simple mathematical model for an island equilibrium

Our model takes a simple analytical equilibrium upon which we superpose a disturbance that generates an island chain. We shall use the toroidal coordinate system (r, θ, ϕ) that relates to the common cylindrical coordinate system (R, Z, ϕ) by $R = R_0 + r \cos \theta$ and $Z = r \sin \theta$.

3.1 An analytic equilibrium

We assume an "equilibrium" as given by profile-consistency considerations [3]: a low- β plasma with circular and concentric flux surfaces. The current density is simply given as:

$$j(x) = j_0 \{1 + q_a x^2\}^{-2}, \quad (1)$$

where $x = r/a$ and a is the minor radius of the plasma. The central current density j_0 is given by $j_0 = I_p(1+q_a)/(\pi a^2)$, where I_p is the total plasma current and q_a the boundary safety factor. The poloidal field is given by:

$$B_\theta(x) = \frac{\mu_0 j_0 a x}{2 \{1 + q_a x^2\}}, \quad (2) \quad q(x) = q_a \left(\frac{1 + q_a x^2}{1 + q_a} \right) \quad (3)$$

Thus, taking $a = 0.165$ m (the RTP value) and $q_a = 2.8$, the $q=2$ surface is located at $r_{mn} = 0.129$ m. The plasma current I_p is calculated from q_a and $B_\phi = 2.1$ T: $I_p = 141$ kA.

The total magnetic field can be written as

$$\mathbf{B} = -B_\phi \mathbf{e}_\phi - \frac{1}{R} \nabla \psi \times \mathbf{e}_\phi \quad (4)$$

where B_ϕ , the toroidal magnetic field and ψ is the usual poloidal flux. For simplicity, we take B_ϕ constant and likewise $R = R_0$, so that our model is effectively cylindrical. Anticipating the definition of a (m,n) magnetic island in Section 3.2, we introduce local coordinates (r, ζ, ξ) :

$$\begin{aligned} \zeta &= m\theta - n\phi && \text{(coordinate across the field lines on an } (m,n) \text{ rational surface) and} \\ \xi &= \frac{nr_{mn}}{R} \theta + \frac{mR}{r_{mn}} \phi && \text{(coordinate along the field lines)} \end{aligned}$$

These coordinates are chosen such that $\nabla \zeta \cdot \nabla \xi = 0$ on the rational surface. An alternative manner of writing the magnetic field is then:

$$\mathbf{B} = \nabla \psi_\zeta \times \nabla \zeta + \nabla \psi_\xi \times \nabla \xi, \quad (5)$$

where the fluxes ψ_ζ and ψ_ξ are, in terms of the quantities of Eq. (4):

$$\psi_\zeta = \frac{1}{g_{mn}} \left(\frac{1}{2} m B_\phi r^2 R / r_{mn} + n \psi r_{mn} / R \right) \quad (6a)$$

$$\psi_\xi = \frac{1}{g_{mn}} \left(\frac{1}{2} n B_\phi r^2 - m \psi \right) \quad (6b)$$

where $g_{mn} = m^2 R / r_{mn} + n^2 r_{mn} / R$. This way of writing the magnetic field has the advantage that $\nabla \psi_\xi$ vanishes on the rational surface, thus simplifying the description of the magnetic field in the vicinity of the rational surface.

From Eqs. (2) and (4) we calculate the main poloidal flux ψ :

$$\psi(x) = \frac{\mu_0 j_0 R_0 a^2}{4 q_a} \ln(1 + q_a x^2) \quad (7)$$

such that ψ_ξ can be written, using Eq. (6b):

$$\psi_\xi = \frac{a^2 B_\phi}{2 g_{mn}} \left(n x^2 - \frac{m(1+q_a)}{q_a^2} \ln(1+q_a x^2) \right) \approx \frac{a^2 B_\phi}{g_{mn}} \frac{n^2}{m} \frac{q_a^2 x_{mn}^2 (x-x_{mn})^2}{1+q_a} + C \quad (8)$$

This flux function contains all the information we need to describe the magnetic field components *perpendicular* to the field line direction on the rational (m,n) surface. The approximation on the r.h.s. of Eq. (8) is to second order in $(x-x_{mn})$. The irrelevant constant C is chosen equal to zero.

3.2 The island

The island, located on the (m,n) rational surface referred to above, is generated by currents predominantly directed along the field lines on the rational surface. Therefore, the island can be described in terms of a single flux function $\tilde{\psi}_\xi$ that gives rise to a radial field perturbation. We arbitrarily define the island flux as

$$\tilde{\psi}_\xi = -\frac{N}{2} (1 + \cos \zeta) e^{-(x-x_s)^2 / \Delta^2} \quad (9a)$$

where x_s is the position of the island ($x_s \approx x_{mn}$), and N and Δ are factors determined by the island width w and the amplitude of the radial field disturbance on x_s . From the approximation of ψ_ξ , Eq. (8), we find

$$N \approx \frac{w^2 B_\phi}{4g_{mn}} \frac{n^2 q_a^2 x_{mn}^2}{m(1+q_a)} e^{(w/(2a\Delta))^2} \quad (9b) \quad \tilde{B}_r(x_s) \approx -\frac{Ng_{mn}}{2Rax_s} \sin \zeta \quad (9c)$$

Note that Eqs. (9b,c) are in exact agreement with White's formula for narrow islands [4], when $w/(2a) \rightarrow 0$. The actual position of the island x_s is chosen by demanding that the force $\mathbf{j} \times \mathbf{B}$ in the O-point of the island be very small. Note that $x_s < x_{mn}$ affects the interpretation of measurements of external field fluctuations due to the island, usually assumed to be generated at $x = x_{mn}$, and thus affects calculations of island widths. From Eq. (9) the perturbed field $\tilde{\mathbf{B}}$ and current density $\tilde{\mathbf{j}}$ may be derived by taking the appropriate derivatives (Eq. (5)).

3.3 Ideal MHD force balance

Ideal MHD equilibria must satisfy $\nabla p = \mathbf{j} \times \mathbf{B}$. Thus, since the total magnetic field can be written, in the presence of the island:

$$\mathbf{B} = \nabla\psi_\zeta \times \nabla\zeta + \nabla(\psi_\xi + \tilde{\psi}_\xi) \times \nabla\xi,$$

and with the plausible assumptions that $p = p(\zeta, r)$ and $\psi_\zeta = \psi_\zeta(r)$, it follows that $p = p(\psi_\xi + \tilde{\psi}_\xi) = p(\Psi_\xi)$.

One may ask to what degree Eqs. (1-3) combined with Eq. (9) actually do constitute an equilibrium. We choose $\tilde{B}_r^{\max} = 7$ mT; $w = 2.9$ cm; $x_s = 0.65$. Eqs. (9b,c) then yield N and Δ and we compute the equilibrium using Eqs. (8) (the exact formula) and (9a). The flux Ψ_ξ is shown in Fig. 1. Because $x_s < x_{mn}$ the island is asymmetric (in accordance with observation, cf. eg. [5]). The radial component of the force $\mathbf{j} \times \mathbf{B}$ is much larger than its toroidal or poloidal components (except in a few internal points of the island), so we may compute $p(r, \theta, \phi)$ to good approximation by radially integrating this force inwards from the plasma boundary. The condition $p = p(\Psi)$ is satisfied to within 30% near the separatrix of the island, the deviation being much smaller (around 10%) in the island interior (Fig.2). The large deviation from the equilibrium condition near the separatrix is inevitable, because the island is embedded in a plasma with a considerable pressure gradient through the X-points.

3.4 Deviation from ideal MHD: plasma flow

As mentioned in the introduction, we proceed to calculate the flow in the island. We assume that some mechanism such as heat transport or mode rotation (cf. [1] and [2]) creates a pressure gradient across the island. More specifically, we impose the pressure profile as it is in the X-point region throughout the plasma. This must be regarded as an extreme case: in reality the mechanisms cited above will never quite be able to establish such a situation, although possibly they may come close.

In absence of ideal MHD equilibrium, the force balance equation must be modified. The steady state in a constant-density magnetofluid is described by (cf. eg. [6]):

$$\mathbf{j} \times \mathbf{B} - \nabla p = \rho \nu \nabla^2 \mathbf{v} \quad (10)$$

where ρ is the density, ν the kinematic viscosity and \mathbf{v} the flow velocity. This equation allows computation of the "flow" $\mathbf{G} = \rho \nu \mathbf{v}$ under the assumption that ν , also, does not depend on space, given boundary conditions. We present preliminary results of this calculation. Fig. 3 displays the radial component of \mathbf{G} in X- and O-point and Fig. 4 shows the flow field. It is observed that the flow is mainly radial and converges close to the inner island separatrix. Possibly the sink can be identified with transport along the field lines that twist around the O-point and connect the high-pressure to the low-pressure region.

4. Conclusions

The semi-analytical island model presented in the present work can lay only limited claim to reality. Nevertheless, it serves as a demonstration of the importance of flow in magnetic island regions. The flow arises when some mechanism creates a pressure gradient across the island. In earlier work candidates for such mechanisms were proposed (e.g. local transport within the island or mode rotation). In general, these mechanisms tend to be effective only at low local temperatures ($T_e < 500$ eV). This flow possibly exerts a significant influence on global radial transport, and it seems necessary to develop ways of estimating the size of this effect. The present work provides only a first attempt, and it is hoped that more realistic models can be constructed in the future.

References

- [1] B.Ph. van Milligen, A.C.A.P. van Lammeren, N.J. Lopes Cardozo, F.C. Schüller, M. Verreck, Proc. 19th Eur. Conf. Innsbruck, 1992, Vol. 16C, Part I, EPS (1992) I-123
- [2] B.Ph. van Milligen, A.C.A.P. van Lammeren, N.J. Lopes Cardozo, F.C. Schüller, M. Verreck, Submitted to Nucl. Fusion 1993
- [3] F.C. Schüller, D.C. Schram, J. Konings, et al., in Contr. Fus. and Plasma Phys. (Proc. 18th Eur. Conf. Berlin, 1991), Vol. 15C, Part IV, European Physical Society (1991) IV-185
- [4] R.B. White and D.A. Monticello, Phys. Fluids **20** (1977) 800 - 805
- [5] J.A. Wesson, R.D. Gill, M. Hugon, et al., Nuclear Fusion **29** (1989) 641-666
- [6] Y.Z. Agim and D. Montgomery, Plasma Physics and Controlled Fusion **33** (1991) 881 - 902

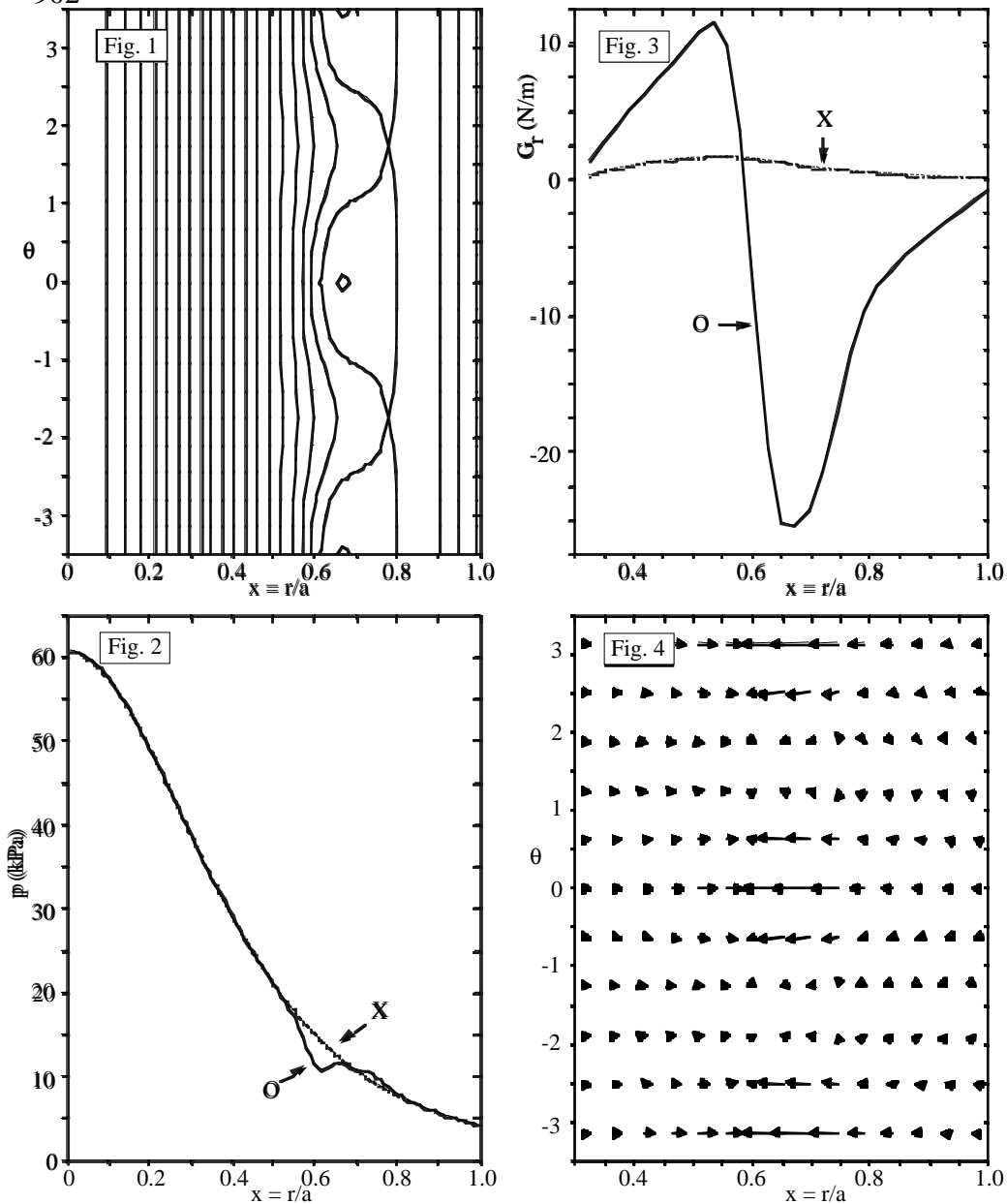


Figure captions

- Fig. 1 Plot of Ψ_ξ -contours in the (x, θ) plane
- Fig. 2 Pressure profile taken through the X- and O-points of the island
- Fig. 3 Graphs of the "flow" G_r through the X- and the O-points
- Fig. 4 Vector plot of \mathbf{G} in the (x, θ) plane