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On the use of toroidal harmonics in the design of magnetic diagnostics

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Abstract

A general method for the optimization of the design of magnetic diagnostics for toroidal fusion devices with a view to maximizing plasma parameter information recovery is presented, based on the combination of plasma equilibrium calculations, expansion of the magnetic field in toroidal harmonics and multivariate regression techniques. As an example, the recovery of $\langle\beta\rangle$ is optimized for the TJ-IU torsatron.

Introduction

Knowledge of the geometry of the magnetic field is fundamental to the understanding of plasma behaviour in all toroidal devices used in fusion research, and essential to the correct interpretation of data from many other diagnostics. It is therefore surprising that in general little effort is put into the systematic design of the magnetic diagnostic in order to extract the maximum information possible about the magnetic geometry. Some work in this direction, enfocused on the interpretation of magnetic measurements rather than the design of the diagnostic itself, has been realized at the W VII-AS stellarator [1].

At the CIEMAT institute, two stellarators are under construction: a torsatron (TJ-IU) and a flexible heliac (TJ-II). The present work presents a generally applicable method for the systematic design of magnetic diagnostics, applied to the TJ-IU torsatron.

Synopsis of the method

The magnetic diagnostic design method is based upon three techniques: (1) Use is made of the stellarator equilibrium code NEMEC [1, 2] to generate a database of equilibria. This database contains a few scans of important plasma parameters (magnetic axis position, β, ...). Of necessity, this is a small database due to the large amount of CPU time needed to calculate each equilibrium. (2) The toroidal harmonics [3] are used to expand the magnetic field outside the plasma. This has three advantages: firstly, it allows us to interpolate between calculated equilibria in a prudent way, by interpolating in moments-space. Thus the database is enhanced considerably; secondly, by specifying the moments rather than the magnetic field in some fixed points, one is still free to choose the positions of e.g. the magnetic pick-up coils (i.e. the enhanced equilibrium database is design-independent); and thirdly, it allows a distinction to be made between dominant low-m and -n effects and indetectable high-mode number effects. (3) Using a fast reconstruction technique known as FP [4] we determine, for a number of magnetic diagnostic designs, how well the plasma parameters can be recovered. Within the limits of the design, given by practical considerations such as cost and available coil mount positions, we find by a minimization process the optimum design corresponding to a maximum of information recovery. The technique is readily applicable to the design of magnetic diagnostics for many types of toroidal fusion devices.

Equilibrium calculations with NEMEC

TJ-IU has the following parameters: l = 1, Toroidal periodicity $N_T = 6$, major radius $R_0 = 0.6$ m, minor radius of the region of interest (the plasma region) $a_{min} = 0.15$ m, inner minor radius of the helical coil $a_{coil} = 0.24$ m. The equilibrium calculations have been performed with the NEMEC-DIAGNO package. NEMEC [1] computes free boundary equilibria using a steepest descent algorithm and assumes nested flux surfaces. DIAGNO [2] calculates the response of the magnetic field to the plasma using data from the equilibrium. As a primary step, only cases with zero toroidal current have been considered.

Determination of moments as a function of plasma parameters

In Ref. [3] an elaborate description is given of how any vacuum magnetic field is expanded in toroidal harmonics (half-integer Legendre functions). Within the limited scope of the present work, we have only investigated the behaviour of the moments as a function of $\langle\beta\rangle$, but emphasize that the simultaneous effect of various plasma parameters can be studied with this method.

Table 1

index	n	m	Description
1	0	0	Toroidal current
2	0	1	$\cos(n\phi)\sin(m\eta)$
3	0	2	"
4	0	3	"
5	0	4	"
6	0	5	"
7	0	6	"
8	6	0	$\sin(n\phi)\cos(m\eta)$
9	6	1	$\cos(n\phi)\sin(m\eta)$
10	6	1	$\sin(n\phi)\cos(m\eta)$
11	6	2	$\cos(n\phi)\sin(m\eta)$
12	6	2	$\sin(n\phi)\cos(m\eta)$
13	6	3	$\cos(n\phi)\sin(m\eta)$
14	6	3	$\sin(n\phi)\cos(m\eta)$
15	6	4	$\cos(n\phi)\sin(m\eta)$
16	6	4	$\sin(n\phi)\cos(m\eta)$
17	6	5	$\cos(n\phi)\sin(m\eta)$
18	6	5	$\sin(n\phi)\cos(m\eta)$
19	6	6	$\cos(n\phi)\sin(m\eta)$
20	6	6	$\sin(n\phi)\cos(m\eta)$
21	12	0	"
22	12	1	$\cos(n\phi)\sin(m\eta)$
23	12	1	$\sin(n\phi)\cos(m\eta)$
24	12	2	$\cos(n\phi)\sin(m\eta)$, etc.

The magnetic field of 5 plasma configurations with $\langle\beta\rangle$ varying between 0 and 1.26% was calculated in a toroidally annular region outside the plasma, of which the inner and outer boundaries coincided with toroidal coordinate surfaces (the toroidal coordinates being defined in Ref. [3]). The inner surface had a minor radius of 0.17 m, and the outer of 1.3 m. The field was calculated on 1000 points that were chosen randomly in this region in order to avoid ill-conditioning of the regression. The field generated by currents inside the plasma was fitted to an expansion in internal harmonics, i.e. without taking into account the external magnetic field caused by the magnetic field coils. From this fit the internal moments were determined along with their errors. Fig. 1 shows the moments at $\langle\beta\rangle = 1.26$ %, the solid line giving the absolute value of the moments and the dashed line the error. The meaning of the horizontal axis, called "index", is explained in Table 1 (here ϕ is the toroidal angle and η is the poloidal angle in the toroidal coordinate system, see Ref. [3]). From Fig. 1 it is seen that for large toroidal mode number n the determination of the moments becomes difficult (the error level becomes comparable to the size of the moments themselves). Likewise, the determination of the moments is of increasing difficulty with increasing poloidal mode number m. This is in accordance with intuition, since higher-order moments decay more rapidly with increasing distance from the plasma and are therefore more difficult to determine.

We therefore focussed on the low-order moments that were well-determined ($n \leq 6$, $m \leq 3$). Fig. 2 shows the dependence on the parameter $\langle\beta\rangle$ of moments that suffered an error less than 30%. It is observed that the lowest-order moment (n=0, m=1) has the strongest variation with $\langle\beta\rangle$, which is related to the displacement of the magnetic axis with $\langle\beta\rangle$.

The variation of each of these 9 moments as a function of $\langle\beta\rangle$ was approximated by third-order polynomials in $\langle\beta\rangle$ (in the case that more than one plasma parameter is varied, a multivariate regression would be necessary at this point). These polynomials were then used to generate the enhanced database of 500 entries, each entry consisting of a randomly selected value of $\langle\beta\rangle$ (in the range $0 \leq \langle\beta\rangle \leq 1.2\%$) and the corresponding moments. The external moments, which were constant for this $\langle\beta\rangle$ scan, were also stored.

Regression analysis

In the present work we have limited ourselves to Mirnov coils that provide point measurements of the magnetic field. Note, however, that other diagnostic elements such as diamagnetic loops can be incorporated in the technique. We define 3 types of coils: radial (r), poloidal (θ), and toroidal (ϕ) field coils that each measure one field component. We use three coils of each of these three types. The radial position of coil i ($1 \leq i \leq 9$) is fixed (r=0.17 m), but the poloidal (θ_i) and toroidal (ϕ_i) positions are variable.

We define a parameter χ^2 which is a function of the coil positions and equals the sum of squares of the reconstruction errors in all parameters p_j that identify the plasma state: $\chi^2(\{\theta_i,\phi_i\},i=1,...,9) = \sum w_j \varepsilon^2(p_j)$, where w_j is some set of weights. In this particular case, we have only one parameter, $p_1 = \langle\beta\rangle$, and we set $w_1 = 1$. The reconstruction error ε in a plasma parameter for a particular choice of coil positions $\{\theta_i,\phi_i\}$ is determined by the method of Function Parametrization (FP) [4], which provides a rapid way of determining a multivariate polynomial approximation of the complex function that gives the plasma parameters ($\langle\beta\rangle$ and others if necessary) in terms of a set of given measurements on the basis of a statistical analysis, i.e. it determines the functions F_j such that $p_j = F_j(\{B_i\},i=1,...,9) + \delta(p_j)$, where the B_i represent the (9) measurements of the magnetic field made by the pick-up coils. The functions F_j are determined by means of a regression minimizing $\delta(p_j)$ from noiseless data, but the reconstruction error $\varepsilon(p_j) = p_j - F_j(\{B_i\},i=1,...,9)$ is calculated by first adding 5% random noise to the simulated measurements [4], to guarantee robustness with respect to noise. χ^2 was first evaluated for some random sets of coil positions, and from the lowest point found a systematic minimization using a quasi-Newton algorithm was carried out. The resulting relative reconstruction error of $\langle\beta\rangle$ with 9 coils was found to have a minimum at 4.4% for the set of coil positios as shown in Fig. 3. However, the principal component analysis which is a part of FP shows that there are only 3 significant principal components, such that in principle 3 well-placed coils would be sufficient to determine $\langle\beta\rangle$.

Thus, the minimization of χ^2 provides the set of coil positions $\{\theta_i,\phi_i\}$ that reduce the reconstruction error in the selected plasma parameters to a minimum, i.e. they maximize the information obtained about these parameters. In the process, the minimum number of measurements needed to determine a (set of) plasma parameter(s) is estimated, which is an important consideration in the design activity.

Conclusions

In the present work we have shown how plasma equilibrium calculations, the expansion of the vacuum magnetic field in toroidal harmonics and multivariate regression techniques may be combined to design a magnetic diagnostic that is optimized to yield the maximum information possible on certain plasma parameters. The intermediate step of the expansion in moments is essential, especially for stellarators, in order to save computing time and storage space, but apart from that allows a clear distinction to be made between low-order dominant effects of the varying plasma parameters and high-order indetectable components. Finally, by evaluating the principal components of the measurements, the minimum number of required measurements is found. Our equilibrium database contains only zero toroidal current cases. Non-zero toroidal currents will affect the coil responses, while it is doubtful that different shaping of the profiles has a large effect on the signals. Future databases are planned to take those aspects into account.

References

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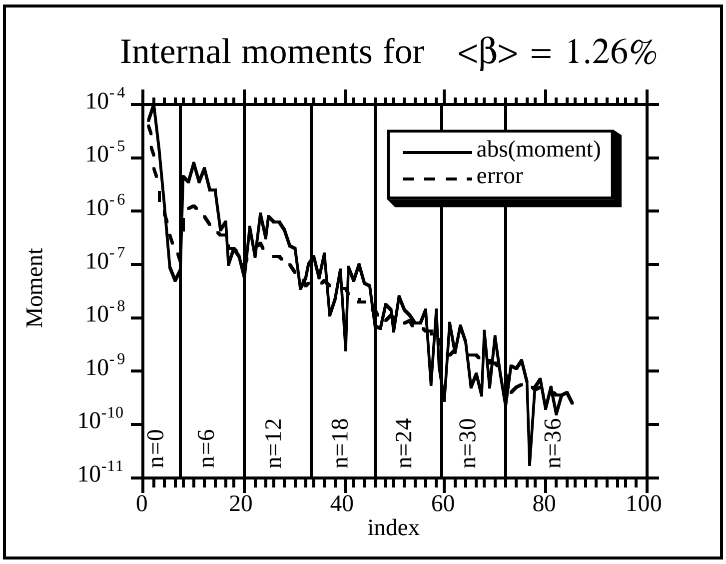

Fig. 1

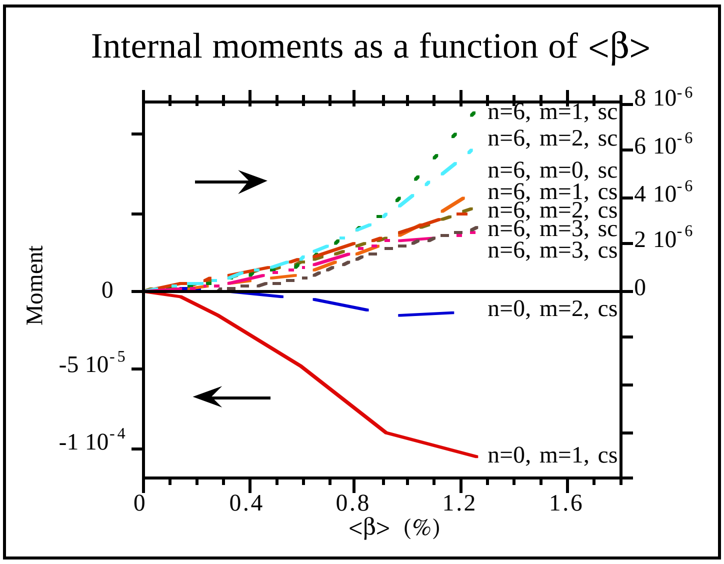

Fig. 2

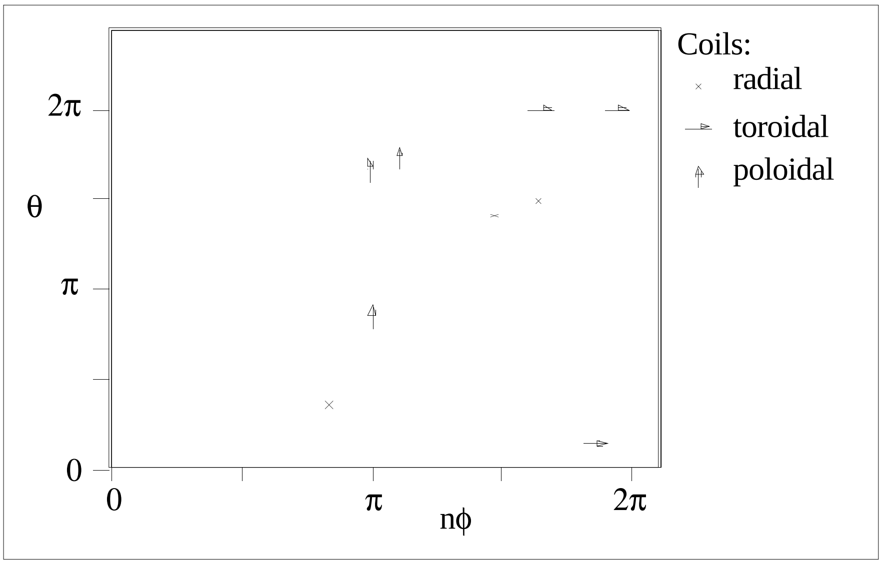

Fig. 3