Magnetic phase transition model for L to H transition

Emilia R. Solano¹, Richard D. Hazeltine²

¹ Laboratorio Nacional de Fusión, Asociación EURATOM-CIEMAT para Fusión, Madrid, Spain

² Institute for Fusion Studies, Univ. of Texas at Austin, Austin, USA

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Plasma magnetization in a tokamak

Plasma force balance:
\[ \nabla p = \vec{j} \times \vec{B} = \vec{j}_\zeta \times \vec{B}_\theta + \vec{j}_\theta \times \vec{B}_\zeta \]

In cylindrical approximation:
\[ \frac{d}{dr} \left( p + \frac{B_z^2 + B_\theta^2}{2\mu_0} \right) = -\frac{B_\theta^2}{\eta\mu_0} \]

Integrating:
\[ \beta_\theta = \int_{0}^{a} \frac{p dS}{B_{\theta a} / 2\mu_0} = \frac{B_z^2 - \langle B_z^2 \rangle}{B_{\theta a}^2} \simeq 1 + \frac{2B_{za}(B_{za} - \langle B_z \rangle)}{B_{\theta a}^2} \]

\( \beta_\theta \) is related to average plasma magnetisation:

\[ \beta_\theta \begin{cases} < 1 & \text{B}_z \text{ increased by } j_\theta \\ \text{paramagnetism, low pressure} \\ > 1 & \text{B}_z \text{ reduced by } j_\theta \\ \text{diamagnetism, high pressure} \end{cases} \]
**Magnetism in cylindrical blob with pressure hill/hole**

\[ F = m n \frac{d\mathbf{v}}{dt} = -\nabla \tilde{p} + j \times \mathbf{B} \]

\[ \tilde{j}_\perp = \frac{\mathbf{b} \times \nabla \tilde{p}}{B} \]

**Diamagnetic current:**
if inside the tube there is a pressure hill, the associated perpendicular current reduces \( B_z \): diamagnetism
**Magnetism in cylindrical blob with pressure hill/hole**

\[ F = m \frac{\partial \mathbf{v}}{\partial t} = -\nabla \tilde{p} + \mathbf{j} \times \mathbf{B} \]

\[ \mathbf{j}_\perp = \frac{\mathbf{b} \times \nabla \tilde{p}}{\mathbf{B}} \]

**Diamagnetic current:**
If inside the tube there is a pressure hill, the associated perpendicular current reduces \( B_z \): **diamagnetism**

**Paramagnetic current:**
If inside the tube there is a pressure hole, the associated perpendicular current increases \( B_z \): **paramagnetism**
Magnetism in cylindrical blob with pressure hill/hole

\[ \mathbf{F} = m n \frac{d\mathbf{v}}{dt} = -\nabla \tilde{p} + \tilde{\mathbf{j}} \times \mathbf{B} = 0 \]

\[ \tilde{\mathbf{j}}_\perp = \frac{\mathbf{b} \times \nabla \tilde{p}}{B} \]

**Diamagnetic current:**
if inside the tube there is a pressure hill, the associated perpendicular current reduces \( \mathbf{B}_z \): diamagnetism

**Paramagnetic current:**
if inside the tube there is a pressure hole, the associated perpendicular current increases \( \mathbf{B}_z \): paramagnetism

**Magnetization of the blob:**

\[ \nabla \times \mathbf{M} = \mu_0 \frac{\mathbf{b} \times \nabla \tilde{p}}{B} = - \frac{d\mathbf{M}}{dr} \hat{r} \]

\[ \tilde{\mathbf{M}} = \frac{1}{\lambda} \int_0^\rho \frac{\mathbf{b}}{B} \frac{\partial \tilde{p}(\rho')}{\partial \rho'} \lambda_\parallel d\rho' \approx -\frac{\tilde{p}}{B} \mathbf{b} \]

\[ \begin{cases} < 0, \text{ dia} \\ > 0, \text{ para} \end{cases} \]
Movement of magnetized object in field gradient

\[ \mathbf{F} = \mathbf{j} \times \mathbf{B} \]

Field gradient created by paramagnetic current in background plasma:
- **Paramagnetic** blobs attracted to high field region.
- **Diamagnetic** blobs attracted to low field region.
Movement of magnetized object in field gradient

(see Jackson)

\[ \mathbf{m}_v \frac{d\mathbf{v}}{dt} \bigg|_V = \int \left( \nabla (\mathbf{M} \cdot \mathbf{B}) \right) dV \approx \nabla \times \nabla \int \int \int \mathbf{v}_M \mathbf{B} r \cdot j \]

\[ \mathbf{B} = \mathbf{B}_0 + r \cdot \nabla \mathbf{B}_0 + \ldots \]

\[ \simeq \left( \int (r \times j_{\text{mag}}) dV \right) \int \nabla B_{0z} dV \]

blob magnetization

\[ \mathbf{m}_n \frac{d\mathbf{v}}{dt} \simeq \mathbf{M}_z \nabla \mathbf{B}_{z0} \]

the cold blob (paramagnetic) seeks high field

the hot blob tube (diamagnetic) seeks low field

Blob averaged \( dB_z/dr \) controls motion of magnetised plasma blobs:

**Anti-potential** leads to *magnetic phase separation*
Paramagnetic plasma: L-mode

Motion of pressure blobs depends on $dB_z/dr$

$$mn_V \frac{d\mathbf{V}_r}{dt} \sim \tilde{M}_\zeta \nabla_r \bar{B}_\zeta$$

paramagnetic cold blobs move inward,
diamagnetic hot blobs move outward

outward thermal energy convection at the expense of
inward magnetic energy convection

$p$ blobs “grow”, “instability”

L-mode
Diamagnetic plasma: H-mode

Motion of pressure blobs depends on $\frac{dB_z}{dr}$

$$m_{n_v} \frac{dV_r}{dt} \sim \tilde{M}_{\zeta} \nabla_r \overline{B}_{\zeta_0}$$

diamagnetic hot blobs move inward, paramagnetic cold blobs move outward

inward thermal energy convection at the expense of outward magnetic energy convection

p blobs “decrease”, “saturation”

H-mode
$\nabla p$ increases somewhere, creating diamagnetic region at plasma edge.
At a magnetic phase boundary blobs of the same type accumulate

diamagnetic blobs (heat) seek wells

paramagnetic blobs seek hills

With multiple blobs moving, $p$ and $B_z$ profiles evolve
Magnetic Boundary: phase transition

\[ \nabla p \text{ increases somewhere, creating diamagnetic region at plasma edge.} \]

\[ B_z(r) \]

Magnetization, of both signs, increases.

Phase transition is self-reinforcing.

Pressure pedestal forms, grows.
Pedestal formation at magnetisation boundary

Assume dashed $B_z(r)$, $p(r)$ initial profiles

Ideal MHD with magnetization force

$$\bar{n}_v m_i \frac{d^2 \xi}{dt^2} = \tilde{M}_\xi \nabla \tilde{B}_{0z}$$

$$\left. \frac{\partial B_z}{\partial t} \right|_M = \nabla \times (\tilde{v}_r \tilde{B}_{0z})$$

$$\left. \frac{3}{2} \frac{\partial p}{\partial t} \right|_M = -\nabla (\tilde{p} \tilde{v})$$

Integrating one temporal step $\Delta t$

Pressure steepens in **diamagnetic** regions

Increases **diamagnetism**

Flattens in **paramagnetic** regions,

Increases **paramagnetism**

**Magnetic phase separation** drives pedestal formation
**Interchange instability**¹

- present when radial force acts equally on electrons and ions
- equivalent to the Rayleigh-Taylor instability in a fluid.
- magnetization gradient acting on magnetized plasma blobs replace “gravitational field” or “curvature”.

\[
\gamma = \sqrt{g \lambda_\perp} = \sqrt{- \frac{1}{m_v} \frac{\tilde{p}}{2B^2} \frac{\partial B_z^2}{\partial r} \lambda_r}
\]

**Magnetization interchange growth faster for high magnetisation, blob amplitude & radius, low field & mass**

¹M.N. Rosenbluth and C.L. Longmire, Annals of Physics, Volume 1, Issue 2, May 1957,120
Suydam criterion for interchange instability


\[
\beta' \left( \frac{Rq}{r_s} \right)^2 \left[ \frac{B^2 \kappa_r}{\mu_0} \right] > \frac{q'^2}{4q^2}
\]

magnetic shear opposes interchange of tubes driven by cylindrical curvature and \( \nabla \beta \)

Generalization:
add magnetization force to cylindrical curvature

\[
\beta' \left( \frac{Rq}{r_s} \right)^2 \left[ \frac{B^2 \kappa_r}{\mu_0} + \tilde{M}_z \frac{dB_{0z}}{dr} \right] > \frac{q'^2}{4q^2}
\]

In magnetically mixed states \( \tilde{M}_z \frac{dB_{0z}}{dr} < 0 \)
magnetisation force adds to curvature, instability,
until the magnetic shear \( q' \) or the variation of \( dB_z/dr \) changes.
As heating is applied, low pressure paramagnetic plasmas have degraded confinement, driven by low $\nabla p$

When sufficient heating is applied, $\nabla p$ grows until zero magnetization is obtained somewhere inside the plasma: $\vec{j}_\theta = 0$

$$\nabla p_0 = \vec{j}_\zeta \times \vec{B}_\theta + \vec{j}_\theta \times \vec{B}_\zeta = 0$$

Estimate critical pressure gradient as

$$\frac{dp_0}{dr} = j\zeta B_\theta = E_{\text{loop}} \eta_{\text{Spitzer}} B_\theta$$

Need database of typical of $\nabla p$, loop voltage, resistivity and $B_\theta$

to test predictions

or measurements of $j_\theta$

Explaining $T_e$ threshold for L-H transition via $\eta_{\text{Spitzer}}$?

and associated pressure gradient threshold
Summary and comments

We presented a first-principles based model of plasma magnetization and magnetic phase transition as the basis for triggering confinement transitions.

The magnetic state of the plasma determines convective motion of high and low pressure blobs.

Paramagnetic plasma regions attract cold blobs, become more paramagnetic.

Diamagnetic plasma regions attract hot blobs, becoming more diamagnetic.

A pedestal structure builds up in the magnetic boundary.

Magnetic boundary defines critical magnetization: $j_\theta = 0 \iff \nabla p = j_\zeta \times B_\theta$

Magnetization force drives the interchange mechanism in closed field line region, similar to interchange in SOL.
Observations and applicability conditions for model:

- In L-mode the plasma is ballooning stable, we are not studying ballooning by another name. The mechanism works in a cylinder.

- High collisionality prevents blob particles from sampling high and low field side, which is necessary so the average $\nabla B_{0z}$ (not local) controls their behaviour. Therefore low $\nu^*$ and $\lambda_\parallel > qR$ are necessary/helpful.

- Background flute-like pressure fluctuations are necessary to seed the blobs. Can be seeded by conventional interchange or other instabilities.

- For $j_\zeta$ and $j_\theta$ to be non-zero at the plasma edge (so $\nabla p \neq 0$) the plasma edge temperature needs to be high enough (resistivity low).

- The pressure must be high enough to allow both positive and negative $\vec{p}$. 
Motion of positive pressure bump (hot field-aligned blob) depends on $\nabla (B_z)$.

As blobs move, magnetisation state is reinforced

$$m n_v \frac{d \mathbf{v}_r}{dt} \sim \tilde{M}_\zeta \nabla \overline{B}_\zeta$$

L mode, paramagnetic region:

- **Cold** blobs move inward, reduce $\nabla p$ increase $B$ in higher $B$ region
- **Hot** blobs move outward, reduce $\nabla p$ decrease $B$ in lower $B$ region.
Motion of positive pressure bump (hot field-aligned blob) depends on $\nabla (B_z)$. As blobs move, blob amplitude increases.

L mode, paramagnetic region:
- Cold blobs move inward, up the pressure gradient, reducing it.
- Hot blobs outward, reducing $p$ gradient.

Net outward energy convection
**H mode: diamagnetic plasma**

Motion of a positive pressure bump (hot field-aligned blob) depends on $\nabla (B_z)$.

$$mn_v \frac{d\vec{v}_r}{dt} \approx \tilde{M}_\zeta \nabla \overline{B}_\zeta$$

**H-mode:**

- Diamagnetic hot blobs are driven inward,
- Paramagnetic cold blobs outward,
- Until blob pressure matches background pressure,

**H mode, diamagnetic region:**

- **Cold** blobs move outward, down the pressure gradient
- **Hot** blobs move inward

\[\Longleftrightarrow\] Net inward energy convection
**H mode: diamagnetic plasma**

Motion of a positive pressure bump (hot field-aligned blob) depends on $\nabla (B_z)$.

\[
\dot{\nabla} \nabla B_z \approx \tilde{M}_\zeta \nabla \tilde{B}_\zeta \circ C
\]

**H-mode:**
- diamagnetic hot blobs move inward,
- paramagnetic cold blobs outward,
- until blob pressure matches background pressure,

**H mode, diamagnetic region:**
- **Cold** blobs move outward, down the pressure gradient
- **Hot** blobs move inward

\[\Rightarrow \text{Net inward energy convection}\]
H mode: diamagnetic plasma

Motion of a positive pressure bump (hot field-aligned blob) depends on $\nabla B_z$.

$$m_n v \frac{d\tilde{V}_r}{dt} \simeq \tilde{M}_\zeta \nabla \tilde{B}_\zeta$$

H-mode:
- diamagnetic hot blobs move inward
- paramagnetic cold blobs outward,
- until blob pressure matches background pressure,
- Net inward energy convection

H mode, diamagnetic region:
- Cold blobs move outward, down the pressure gradient
- Hot blobs move inward

Magnetism in cylindrical blob with pressure hill/hole

\[ \mathbf{F} = mn \frac{d\mathbf{v}}{dt} = -\nabla \tilde{p} + \mathbf{j} \times \mathbf{B} \]

\[ \mathbf{j}_\perp = \frac{\mathbf{b} \times \nabla \tilde{p}}{B} \]

Diamagnetic/paramagnetic current:
if inside the tube there is a pressure hill/hole,
the associated perpendicular current reduces/increases \( B_z \): magnetization is
negative, diamagnetism
positive, paramagnetism

Magnetization of the blob:

\[ \nabla \times \mathbf{M}_z = \mu_0 \frac{\mathbf{b} \times \nabla \tilde{p}}{B} = -\frac{dM_z}{dr} \hat{r} \]

\[ M_z = -\frac{1}{\lambda_\parallel} \int \mu_0 \mu_0 \frac{\mathbf{b} \times \nabla \tilde{p}}{B} \, dr = -\mu_0 \frac{\Delta \tilde{p}}{B} \]

\[ \begin{cases} < 0, \text{ dia} \\ > 0, \text{ para} \end{cases} \]
Notice that everything here is “stable”:

The time evolution of the fluctuations is such as to reduce their amplitude (in their own frame of reference)

Nevertheless, energy flows are organised by plasma magnetisation and phase separation