Full orbit simulations of impurity transport in spherical tokamaks with strongly-sheared electric fields

C. G. Wrench\textsuperscript{1}, E. Verwichte\textsuperscript{1}, K. G. McClements\textsuperscript{2}

\textsuperscript{1} Centre for Fusion, Space and Astrophysics
University of Warwick
Coventry, CV4 7AL, UK

c.g.wrench@warwick.ac.uk

\textsuperscript{2} EURATOM/CCFE Fusion Association
Culham Science Centre
Abingdon, Oxfordshire, OX14 3DB, UK

TTG Conference, Córdoba
7th September 2010
Introduction

- Advanced tokamak operating scenarios use strongly sheared electric fields to suppress turbulence (Chen 2001)
- In spherical tokamaks (STs), such as MAST, radial electric fields associated with edge transport barriers can vary on length scales of the order of the ion Larmor radius (Meyer, 2009; Shaing, 1998)
- Standard neoclassical theory not valid
- We use full-orbit test-particle approach to study collisional impurity transport in vicinity of transport barriers in STs

Radial electric field as measured in the edge of MAST (Meyer, 2009)
Radial E-field: effect on collisionless orbits

- Impact of a sheared radial electric field on collisionless orbits is to:
  - Increase trapped particle fraction
  - Reduce orbit width of trapped ions
- Orbit squeezing shown by Shaing et al. (1998) to affect neoclassical heat conductivity
- **Collisional particle** transport also affected

Demonstration of orbit squeezing for an Ar\textsuperscript{12+} ion.
Test Particle Simulation

- CUEBIT: Full orbit test particle simulation code
- Solves the Lorentz-Langevin equation:

\[ m_z \frac{dv}{dt} = Ze(E + v \times B) - \frac{m_z}{\tau}(v - u_f) + m_za(t) \]  

\text{Lorentz}\quad\text{Langevin}  

using an implicit finite difference method

- Solves test particle \((Z^2n_z \ll n_i)\) trajectories for a prescribed MAST-like equilibrium
- All fields and bulk ion profiles prescribed and assumed static
- Electrostatic potential prescribed and of the form:

\[ \Phi = \Phi_0 \arctan \left( \frac{\psi - \psi_1}{\Delta \psi} \right) \]  

\text{(2)}

- We assume that the dominant terms in the bulk ion force balance at the transport barrier are \(\nabla p\) and \(ZeE_r\)
Simulation Results: $\alpha$-particles

Simulation of $10^4$ $\alpha$-particles without, (a) and (b), and with, (c) and (d), a sheared radial electric field.
Simulation Results: Ne$^{10+}$ particles

Simulation of $10^4$ Ne$^{10+}$-particles without, (a) and (b), and with, (c) and (d), a sheared radial electric field.
Simulation Results: $W^{20+}$ particles

Simulation of $10^4 W^{20+}$-particles without, (a) and (b), and with, (c) and (d), a sheared radial electric field.
Simulation Results

Poloidal distribution of $10^4$ impurity ions after 100 ms. Red line indicates location of peak radial electric field.
Scaling with particle parameters

Consider collisional equation of motion (Langevin equation):

\[
m_z \frac{dv}{dt} = Ze (E + v \times B) - \frac{m_z}{\tau} (v - u_f) + m_z a(t) \tag{3}
\]

To make analytics tractable, we assume Cartesian geometry and \( E = E_x = \text{const.} \). Solve

\[
\frac{dv_x}{dt} + \nu v_x = \frac{Ze}{m_z} E_x + \Omega v_y \tag{4}
\]

\[
\frac{dv_y}{dt} + \nu v_y = -\Omega v_x \tag{5}
\]

to find:

\[
v_x = v_\perp e^{-\nu t} \sin \Omega t + \frac{E_x}{B} \frac{\nu}{\Omega} \frac{1}{1 + \nu^2/\Omega^2} \tag{6}
\]

\[
v_y = v_\perp e^{-\nu t} \cos \Omega t - \frac{E_x}{B} \frac{1}{1 + \nu^2/\Omega^2} \tag{7}
\]

Note: \( x \)-direction is a proxy for the radial direction
Scaling with particle parameters: Drag force drift

Drag force drift in the same direction as the electric field:

\[
v_x = v_\perp e^{-\nu t} \sin \Omega t + \frac{E_x \nu}{B \Omega} \frac{1}{1 + \nu^2/\Omega^2}
\]  

(8)

This implies that there is:

• an inward-pinch in the presence of a negative radial electric field \((E_x < 0)\) and

• an outward-pinch in the presence of a positive electric field.

In tokamaks we have that \(\nu^2 \ll \Omega^2\), therefore, **drag force drift** is

\[
v_{\text{drag}} \simeq Z \frac{E_x m_i^{1/2} e^3 n_i \ln \Lambda}{6\sqrt{2\pi}^{3/2} \epsilon_0^{2} T_i^{3/2} B^2}
\]

(9)
Scaling with particle mass

Above: Poloidal distributions for (left to right) Ne$^{10+}$, Ar$^{10+}$, Mo$^{10+}$ and W$^{10+}$.

Left: Density profiles in minor radius for impurities with varying masses but the same charge state (same species as above).
Characterising transport: Diffusion coefficients

Diffusivities are estimated from simulated particle flux and density gradient:

\[
D_{\text{eff}} = -\frac{\Gamma_z}{dn_z/dx} \bigg|_{E=E_{\text{max}}} \tag{10}
\]

Assuming particle flux is composed of usual diffusion and pinch terms, we may relate drag force drift to transport:

\[
\Gamma_z = -D \frac{dn_z}{dx} - vn_z \tag{11}
\]

implies

\[
D_{\text{eff}} = \frac{D}{\frac{n_z v}{\Gamma_z} + 1} \tag{12}
\]

where we take \( v \) to be drag force drift velocity at peak of electric field. Therefore:

\[
\frac{D_{\text{eff}}}{D} = \frac{1}{1 + \alpha Z} \tag{13}
\]
Scaling of effective diffusivities with particle mass and charge

- Normalised effective diffusion coefficient for various impurities species, all with $Z=10^+$. Diffusion coefficients are normalised to measured coefficient in the absence of an electric field.

- Normalised effective diffusion coefficient for various charge states of neon.
Scaling with electric field parameters

Variation of effective diffusion coefficient with change in electric field width and strength for $C^{6+}$ ions.

Scaling is of the form:

$$D_{\text{eff}} = D_0 \left( \frac{\Delta r}{\Delta r_0} \right)^{b_1(E-E_0)}$$

$$= 9.436 \left( \frac{\Delta r}{0.159 \rho_L} \right)^{0.121 E} \text{ m}^2 \text{ s}^{-1}$$

Here $\Delta r$ is the electric barrier width in meters and electric field strength, $E$, is measured in kV m$^{-1}$.
Comparison of Carbon Profiles with Experiment

Carbon density profile in MAST as measured by the Charge Exchange Diagnostic. Image courtesy J. McCone.

Scaled $^{6+}$ density profile as computed by CUEBIT.
Concluding remarks

- A strongly sheared electric field can strongly modify collisional transport
- We have used full-orbit test-particle approach to study collisional impurity transport in spherical tokamaks with strongly-sheared electric fields
- Collisional drag in presence of radial $E < 0$ produces local inward pinch, which increases with $Z$ but is independent of impurity mass
- Negative implication of this is the retention of highly ionised species, e.g. tungsten
- Effective diffusion scales with the electric field width with a power proportional to the electric field strength
References